

Computer-Aided Determination of the Natural Vibration Frequency and Logarithmic Damping Decrement of a Mechatronic Cutting Tool*

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Abstract

The article presents a computer-aided method for determining the natural vibration frequency and the logarithmic decrement of damping of the tested object (mechatronic boring tool prototype). The MATLAB environment from MathWorks® and LabView™ from National Instruments was used for this purpose. The conducted research aimed to determine the resonant frequencies of the free vibrations of the tool body and the value of the logarithmic decrement of damping in order to determine their value and the nature of changes depending on the point of attachment of the mechatronic cutting tool in the machine tool holder. The method of data analysis and processing and the results of computer calculations were presented, and then conclusions were formulated.

Keywords: computer science, vibration analysis, mechatronic tools, cutting process

Introduction

In manufacturing processes, there is a constant trend towards improving the accuracy of manufactured items, reducing the number of necessary operations, shortening the processing time and reducing production costs. In the case of machining, one way to introduce improvements is to use mechatronic tools, which are tools equipped with an electronically controlled mechanism or mechanisms. They are therefore a synergy of mechanics, electronics and a control algorithm. The block diagram is presented in Figure 1.

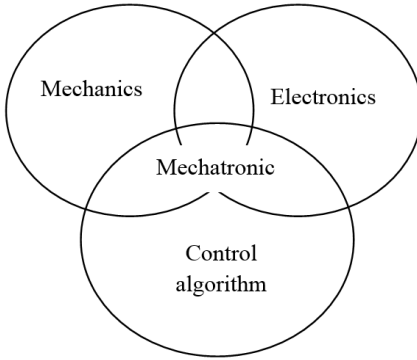


Fig 1. Mechatronics as a combination of mechanism, electronics and control algorithm

This term is often misused by manufacturers, who call tools controlled strictly mechanically mechatronic tools. According to the book by Cichosz and Kuzinovski (2016) mechatronic tools are used, among others, in machining in the processes of turning, boring, milling, grinding and drilling. As a result of increasing technological progress, the requirements for the precision of machine tools and the tools themselves are increasing [1]. One of the challenges facing designers is to improve the dimensional and shape accuracy of workpieces while reducing the number of machining passes. The rigidity of the MCWT (machine tool-chuck-workpiece-tool) system is one of the most important features that determine the final quality of the workpiece. Achieving the intended goals requires overcoming various obstacles. As Kosmol and Wilk (2008) claim in their study this problem that has a significant impact on the final effect of machining is the elastic deformation of the MCWT system [2]. Tool deformations in the turning process are presented in Figure 2. Radial deformations are particularly important from the point of view of machining error, because the diameter error caused by them is doubled. This results from the kinematics of the turning or boring process.

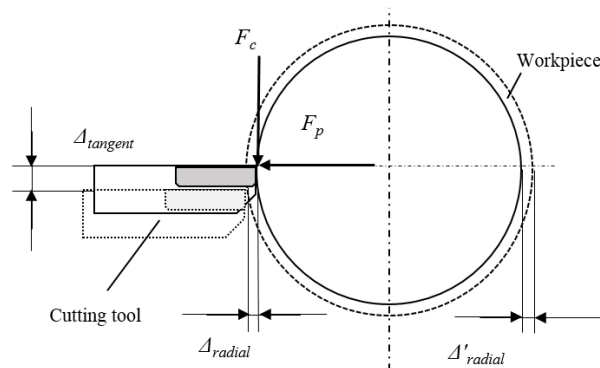


Fig 2. Sources of formation and nature of deformations of the cutting tool F_c – cutting force, F_p – resistive force, Δ_{radial} – radial deflection, $\Delta_{tangent}$ – tangent deflection

The subject of the research was a prototype of a mechatronic tool used for boring deep holes. As Żebrowski writes in his book, a deep hole is one whose length exceeds five diameters [3]. The boring tool used for this purpose must be long enough to perform machining on a given hole length and have a small enough diameter to fit in it. The geometry of the hole determines the geometry of the tool, which is often reduced in comparison to similar types of external turning tools. As a result, the tool becomes more susceptible to elastic deformations during machining, which in turn results in dimensional and shape errors of the workpiece. One way to improve this situation is to use mechatronic tools with real-time measurement and correction of these deformations. Such tools were presented in their studies by, among others, Min (2002), Chiu (2002), Chan (1997) [4-7].

Characteristics of the Tested Mechatronic Tool Prototype

The idea of the developed tool is to optoelectronically measure its deformation in real time and introduce a corrective displacement of the cutting blade using a piezoelectric actuator and equalization of influence of tool small stiffness and non-uniform machining allowance on cutting accuracy. This tool was described in more detail in the work of Cichosz et al. (2014) [8]. The essence of the measurement is presented in Figure 3, a general view of the tool prototype in Figure 4, and the essence of performing corrective displacements in Figure 5.

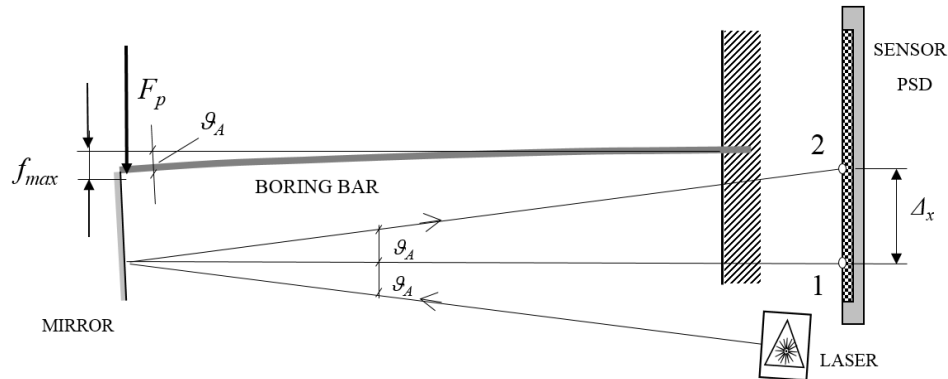


Fig 3. The principle of determining the path of displacement of the light spot on the surface of the PSD matrix in a system mounted on a boring bar: point 1 - undeflected tool, point 2 - deformation under the influence of cutting force F_p

To measure the deformation of the tool, a PSD optoelectronic sensor was used, illuminated by laser light. A beam of light runs in a shielded path along the tool and after reflection from the mirror falls on the surface of a photosensitive PSD diode. Deformation of the tool causes the light to move on its surface.

The developed mechatronic tool was designed in such a way that it could be mounted on a variable overhang of the body. This type of solution allows for conducting tests in various cutting conditions, including in particular in conditions where self-excited vibrations may occur. A series of tests was conducted to determine the resonance frequencies of free vibrations of the tool body and the value of the logarithmic damping decrement in order to determine their value and the nature of changes depending on the point of mounting the tool in the machine tool holder. Vibration tests were conducted in both the tangential and radial directions.

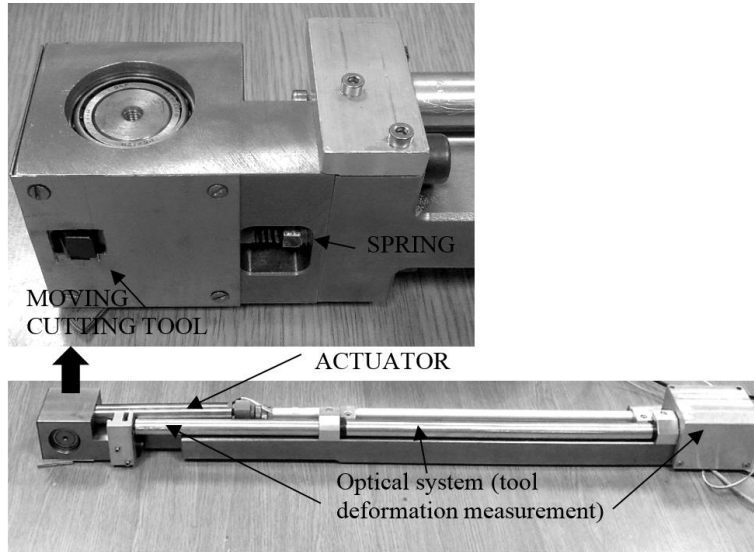


Fig 4. Developed prototype of a mechatronic tool

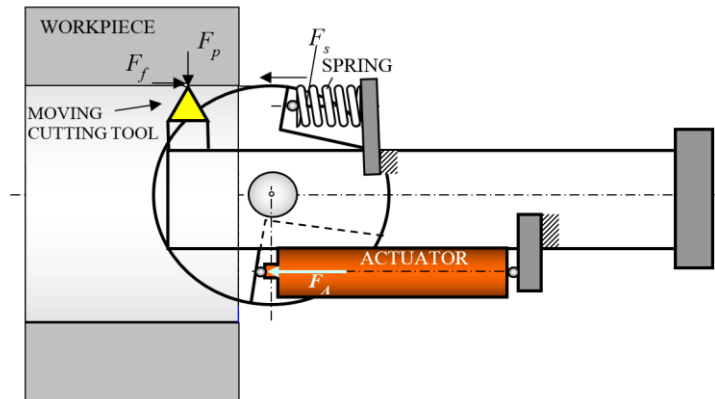


Fig 5. Distribution of forces acting on the rotating element of a mechatronic tool

The most important component, from the point of view of its influence on the machining accuracy, is the resistance force F_p , however, from the point of view of the principle of operation of the designed tool, the feed force F_f also plays an important role. It affects the spring, which eliminates the clearances between the rotating insert, on which the cutting blade is located, and the actuator. Insufficient value of the tensioning force F_s coming from the spring in relation to the feed force may result in uncontrolled rotation of the insert with the blade. For the assumed thickness of allowances and cutting parameters, the actuator must overcome, taking into account the proportions of the arms on which the individual forces act (Figure 5), the total value of the counterforce and the spring pressure force reduced by the value of the feed force. This relationship can be written as follows

$$F_A r_A > F_p r_p + F_s r_s - F_f r_f \quad (1)$$

where:

$F_{A,p,f,s}$ - forces, respectively: actuator, thrust, feed and spring,

$r_{A,p,f,s}$ - arms on which the forces act, respectively: actuator, thrust, feed and spring.

Determination of Resonance Frequencies and Logarithmic Decrement of Damping of the Tested Object in the MATLAB Environment

To determine the logarithmic decrement of vibration damping two methods are used. In the first method, the basis is the ratio of the absolute values of two successive vibration amplitudes of the tested object, which can be written as follows

$$\delta = \ln \frac{A_n}{A_{n+1}} \quad (2)$$

and in the second method, the ratio of amplitudes spaced one period apart

$$\delta_T = \ln \frac{A_n}{A_{n+2}} \quad (3)$$

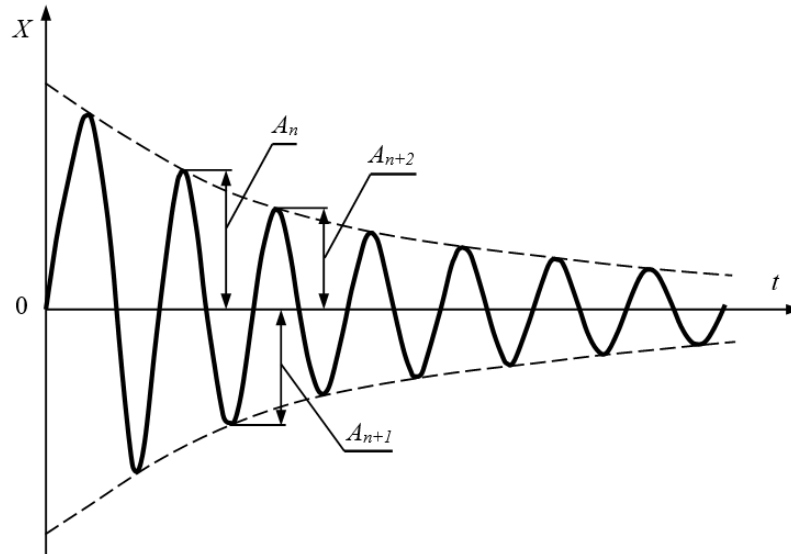


Fig 6. Time history of free vibrations of a linear object with one degree of freedom by Osiński (1997) [9]

In his book, Osiński (1997) proposes an alternative method for determining the value of vibration damping is to determine the relative vibration energy dissipation coefficient ψ . It is defined as the ratio of the value of energy dissipated during one vibration period ΔW to the value of energy supplied during that period W [9]

$$\psi = \frac{\Delta W}{W} \quad (4)$$

For a system with linear elasticity, there is a relationship between this coefficient and the damping decrement. The envelope of the $A=A(t)$ graph of damped vibrations shown in Figure 6 defines the energy in the system, which can be defined as follows

$$W_t = k \frac{A^2(t)}{2} \quad (5)$$

where: k - stiffness coefficient of the system

and therefore

$$\psi = - \int_{W_t}^{W_{t+T}} \frac{dW}{W} = - \int_{A_n}^{A_{n+2}} 2 \frac{dA}{A} = 2 \ln \frac{A_n}{A_{n+2}} = 2\delta_T \quad (6)$$

Appropriately

$$\psi = 2(\delta_1 + \delta_2) \quad (7)$$

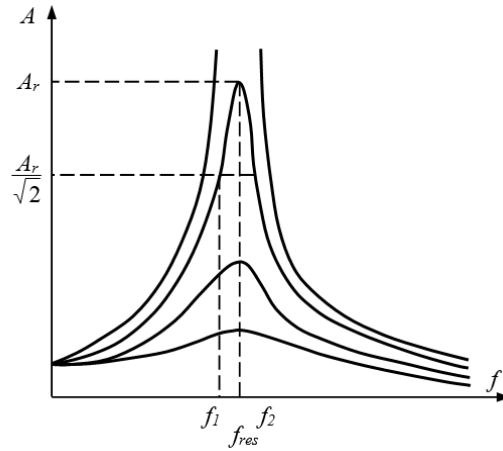


Fig 7. The adopted method of determining the parameters for calculating the damping decrement for the determined amplitude characteristics

The above-mentioned coefficients can be determined from the resonance curve by determining the frequencies f_1 and f_2 for which the signal amplitude is $\sqrt{2}$ smaller than the peak amplitude (Fig. 7). On this basis, the loss coefficient can be determined

$$\eta = \frac{f_2 - f_1}{f_{res}} \quad (8)$$

The following relationship exists between the loss coefficient η and the logarithmic decrement δ of the damping

$$\delta = \pi \frac{f_2 - f_1}{f_{res}} = \pi \cdot \eta \quad (9)$$

The above method was used to determine the tool damping decrement for five different mounting points of the mechatronic tool prototype. In addition, the resonance frequencies of the tool's natural vibrations were determined. In order to determine the above parameters, a laboratory stand was created, as shown in Figure 8.

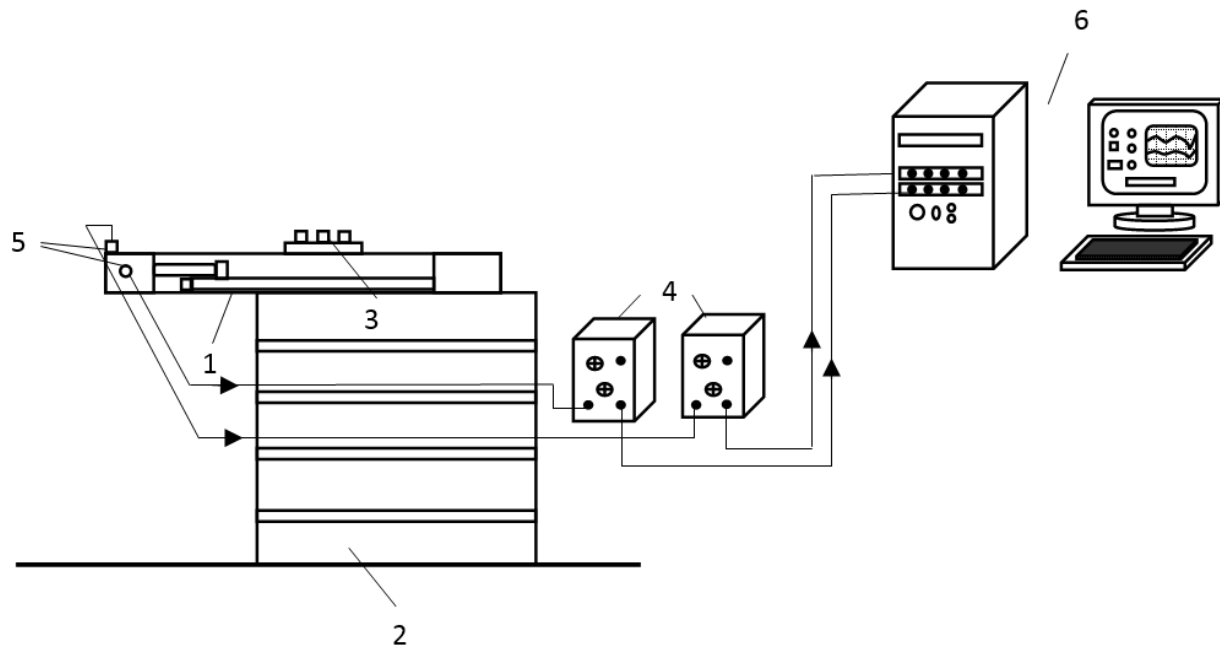


Fig. 8. Schematic diagram of the stand for determining the natural frequency and logarithmic decrement of damping of a mechatronic tool. Components: 1 – mechatronic tool, 2 – base, 3 – tool mounting point, 4 – Brüel & Kjær 2635 amplifiers, 5 – Brüel & Kjær 4384 accelerometers, 6 – PXI-1000 computer with PXI-4472 measuring card

The tested tool was attached to a steel block in a manner similar to its attachment in the machine tool holder. Brüel & Kjær 4384 piezoelectric accelerometers with Brüel & Kjær 2635 amplifiers were used for vibration measurements. These sensors recorded the tool vibration course for an impulsive excitation applied by means of an impulsive hammer. Then the signal from the accelerometers was transferred to a PXI-1000 PC measuring computer with a PXI-4472 measuring card working in the LabView™ environment from National Instruments. The computer recorded measurement signals with a sampling frequency of $f_p=10$ kHz, saving the measurement data to text files. The process of determining resonance frequencies and damping decrement was automated by using a script written in MATLAB, which analyzed data from files created during individual measurement tests, subjecting them to FFT (Fast Fourier Transform) analysis. According to Owen's book (2021) the Fourier transform is widely used in the analysis and synthesis of electronic circuits and systems, as it allows the combination of two ways of representing signals - time-domain representation and frequency-domain representation [10]. Such a transformation allowed obtaining graphs in the frequency domain, which are legible and easy to interpret and allow determining the parameters being tested. They create the so-called spectral characteristics of the tested object. MATLAB script also creating this spectral graphs of resonance curves and providing values of the searched parameters. The tests were carried out for tool overhangs of <230; 280; 330; 380; 430 mm>, performing four tests for each of them. Excitation and measurements were performed in two directions: radial and tangential. The MATLAB script used to determine the resonance frequency of the tested object and the logarithmic damping decrement, along with explanations of its individual functions, is presented below:

```
clear all;           % clear all variables from the current workspace
clc                 % clear all the text from the Command Window, resulting in a clear screen
clf                 % clear the surface plot from the figure and reset all figure properties to their default
values
```

```

fp=10000;           % sampling frequency [Hz]
pion = load('pom48.lvm'); % choosing of data measurement source file (in this example „pom48.lvm”)
pion = pion(:,2);   % choosing second column in source file with measurement data from accelerometer
pion = detrend(pion); % removing trend (offset)
w = abs(fft(pion)); % FFT (Fast Fourier Transorm) analysis
n=length(w);       % vector „w” length
z=zeros(1,n);
%for i=3:n-2           % optional
% z(i)=sum(w(i-2:i+2))/5; % median
%end                  % filter
z=w;
len = length(z);    % vector „z” length
f = linspace(0,fp,len); % frequency domain

Z=max(z);           % returns the maximum value of variable "z"
Z=Z/sqrt(2);
idx=[];

for i=1:(round(len/2)), % start of the characteristics to be analyzed
    idx=[idx i];
end
end
grid on;            % enable grid on chart

plot(f(100:3000),w(100:3000)) % creating a graph in the frequency range from 0 to 300 Hz
dol=f(min(idx));
gora=f(max(idx));
delta_f=gora-dol;
mak=max(z);

id=[];              % parameter calculations
for i=1:len/2;
    if z(i)>=(mak-0.1)
        ww=i;
    end
end
sigma_t=pi*delta_f/(f(ww));

disp('Analysis results:') % displaying calculation results
disp(['Lower frequency limit f1: ' num2str(dol)]) % Lower frequency limit  $f_1$ 
disp(['Upper frequency limit f2: ' num2str(gora)]) % Upper frequency limit  $f_2$ 
disp(['Resonance frequency f_res: ' num2str(f(ww))]) % Resonance frequency  $f_{res}$ 
disp(['delta (difference): ' num2str(delta_f)]) % delta (difference)
disp(['Damping: ' num2str(sigma_t)]) % logarithmic decrement  $\delta$  of the damping

```

Figure 9 shows an example of the script execution result displayed in the MATLAB command line. Additionally, the graphs presented in Figures 10-14 were generated.

```

>> poziom_08
Analysis results:
Lower frequency limit f1: 112
Upper frequency limit f2: 113.5
f_res: 113
delta (difference): 1.5
Damping: 0.041703
fx >>

```

Fig 9. Example results of calculations performed by the presented script in the MATLAB environment

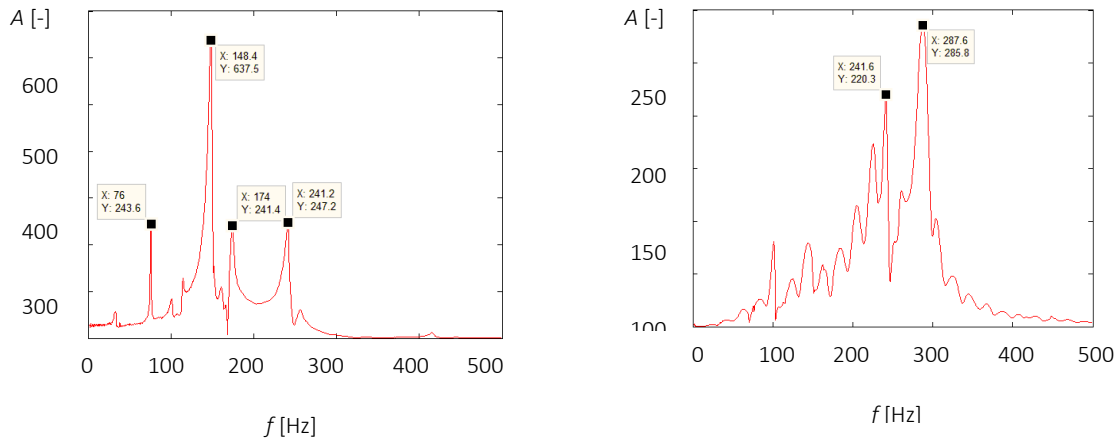


Fig 10. Frequency spectrum of tool natural vibrations in radial (left) and tangential (right) directions for a 230 mm overhang

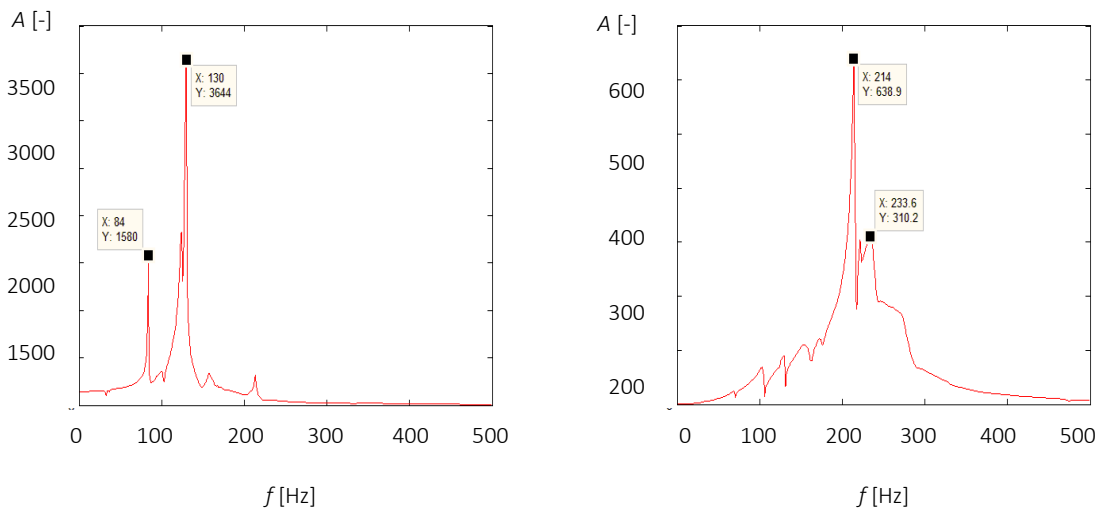


Fig 11. Frequency spectrum of tool natural vibrations in radial (left) and tangential (right) directions for a 280 mm overhang

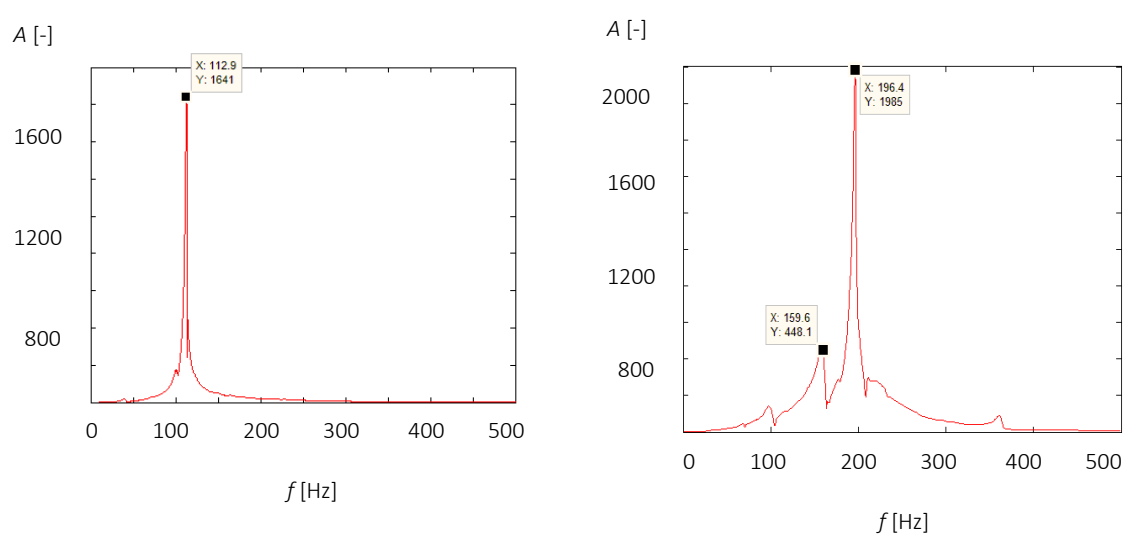


Fig 12. Frequency spectrum of tool natural vibrations in radial (left) and tangential (right) directions for a 330 mm overhang

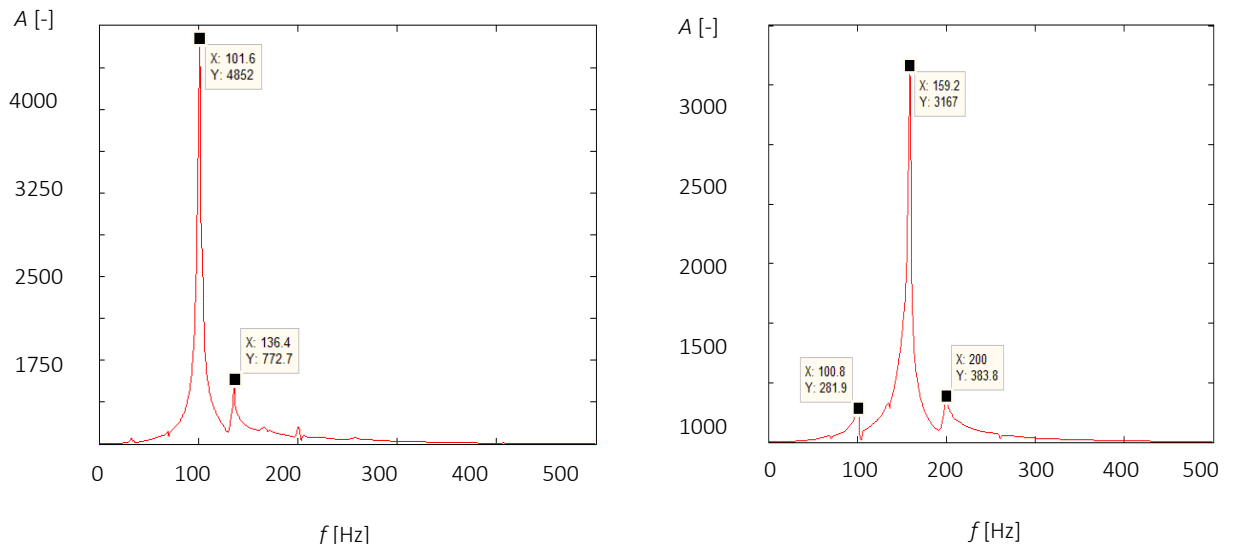


Fig 13. Frequency spectrum of tool natural vibrations in radial (left) and tangential (right) directions for a 380 mm overhang

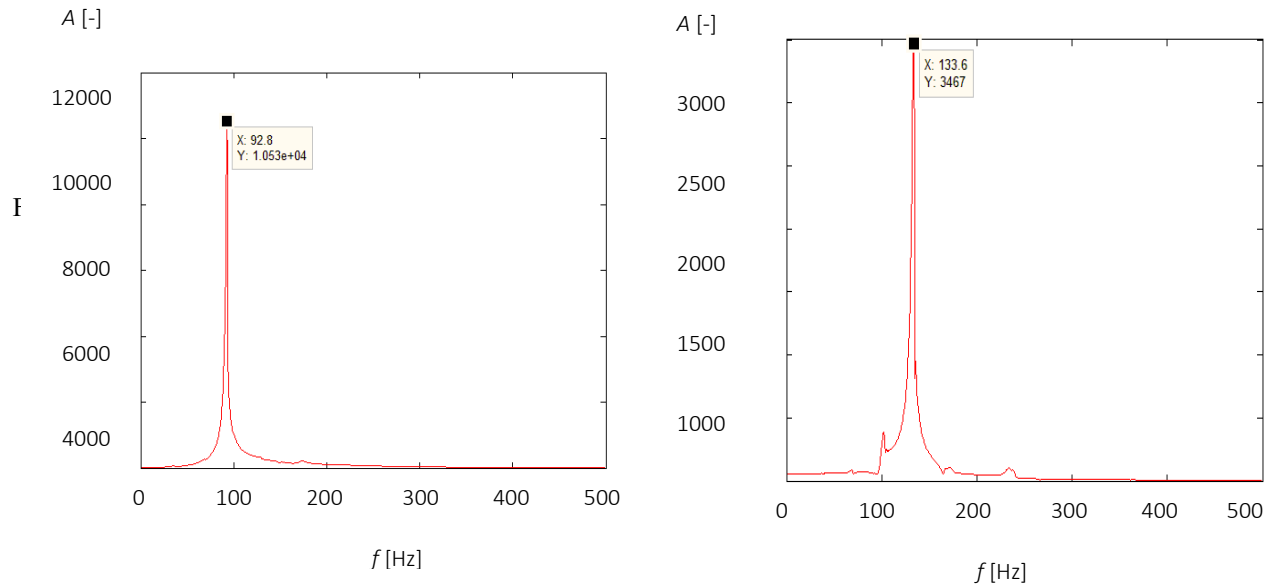


Fig 14. Frequency spectrum of tool natural vibrations in radial (left) and tangential (right) directions for a 380 mm overhang

The obtained results of measurements of the natural vibration frequencies of the tested object are presented in Tables 1 and 2. The results obtained in subsequent measurement tests were repeatable, which is illustrated by the values of standard deviations and confidence intervals for the coefficient $\alpha = 0.05$.

Table 1. Summary of the measurement results of the tool resonance frequency in the tangential direction for its various mounting points

Overhang	Resonant frequency f_{res} in tangential direction [Hz]					Standard deviation	Confidence interval
	[mm]	Measurement 1	Measurement 2	Measurement 3	Measurement 4		
230	292	287,6	290,3	289,4	289,8	10,09	16,05
280	214,2	214,1	214,2	214	214,1	0,03	0,04
330	196,4	196,2	196,3	196,4	196,3	0,03	0,04
380	159,2	159,2	159	159,1	159,1	0,03	0,04
430	133	133,4	133,2	133,1	133,2	0,09	0,14

Table 1. Summary of the results of the tool resonance frequency measurements in the radial direction for its various mounting points

Overhang [mm]	Resonant frequency f_{res} in radial direction [Hz]					Standard deviation [Hz]	Confidence interval [Hz]
	Measurement 1	Measurement 2	Measurement 3	Measurement 4	Average value		
230	144,8	145	145,8	145,2	145,2	0,56	0,89
280	129,6	129,8	129,7	129,8	129,7	0,03	0,04
330	112,6	112,8	112,6	112,9	112,7	0,07	0,11
380	101,4	101,3	101,4	101,3	101,4	0,01	0,02
430	93	92,8	92,9	93	92,9	0,03	0,04

The resonance frequency variation patterns with increasing tool reach are presented in Figs. 15 and 16.

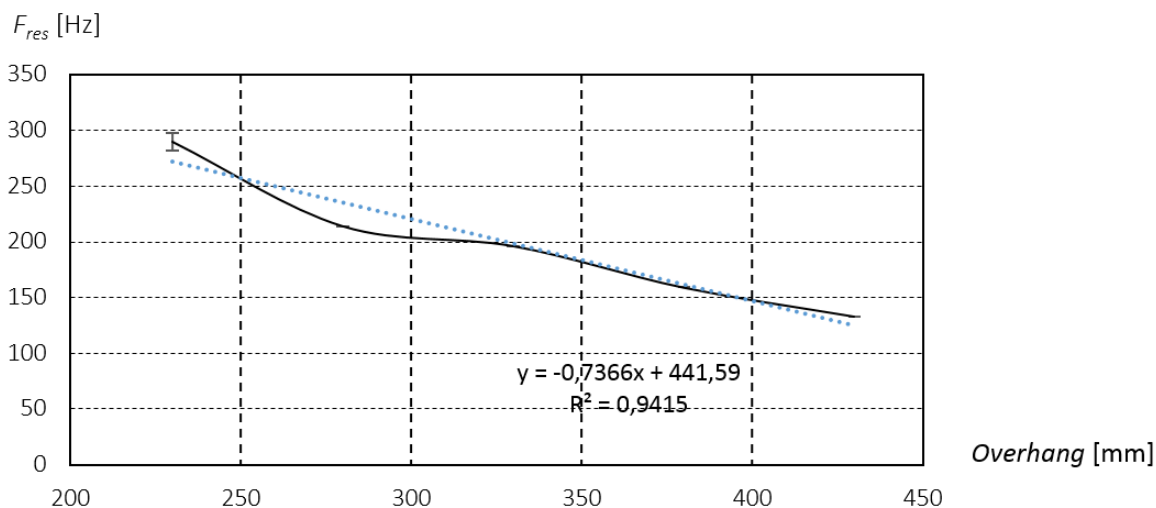


Fig 15. Change of the resonance frequency of the tool's natural vibrations in the tangential direction depending on its overhang

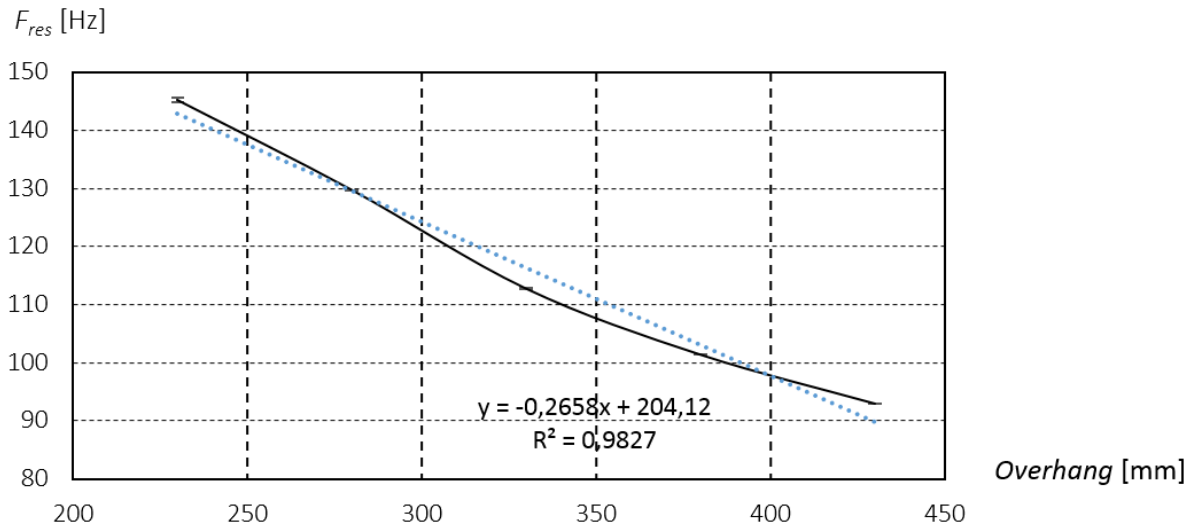


Fig 16. Change of the resonant frequency of the tool's natural vibrations in the radial direction depending on its overhang

For both tangential and radial excitations, it can be assumed that these frequencies change linearly. For the determined trend line, the coefficients of determination, which is the square of the correlation coefficient, at a level above 0.94 indicate very good agreement with the adopted linear model. As the tool overhang increases, the resonance frequency decreases from 290 to 133 Hz in the tangential direction and from 145 to 93 Hz in the radial direction. The resonance bands are characterized by a high steepness and narrow width, so it can be assumed that the tool body is characterized by a low damping value. Tables 3 and 4 summarize the values of the logarithmic decrement for individual measurement tests, obtained as a result of calculations carried out in the MATLAB program.

Table 3. Summary of measurement results of the logarithmic decrement of tool damping in the tangential direction for its various mounting points.

Overhang [mm]	Logarithmic decrement of damping δ in the tangential direction [-]					Standard deviation [-]	Confidence interval [-]
	Measurement 1	Measurement 2	Measurement 3	Measurement 4	Average value		
230	0,245	0,248	0,251	0,263	0,252	1,9E-04	3,02E-04
280	0,059	0,056	0,056	0,058	0,057	5,96E-06	9,49E-06
330	0,048	0,048	0,048	0,048	0,048	1,15E-08	1,82E-08
380	0,055	0,055	0,055	0,055	0,055	0,89E-08	1,42E-08
430	0,061	0,071	0,070	0,062	0,066	75,03E-06	1,19E-04

Table 2. Summary of measurement results of the logarithmic decrement of the tool damping in the radial direction for its various mounting points

Overhang [mm]	Logarithmic decrement of damping δ in radial direction [-]					Standard deviation [-]	Confidence interval [-]
	Measure- ment 1	Measure- ment 2	Measure- ment 3	Measure- ment 4	Average value		
230	0,194	0,217	0,209	0,200	0,205	3,07E-04	4,87E-04
280	0,058	0,058	0,058	0,058	0,058	0,83E-08	1,32E-08
330	0,050	0,059	0,052	0,058	0,055	0,56E-04	8,85E-05
380	0,068	0,068	0,068	0,068	0,068	0,84E-08	1,33E-08
430	0,047	0,041	0,045	0,042	0,044	2,80E-05	4,46E-05

As it results from the presented results, the highest value of the logarithmic damping decrement occurs for the shortest tool overhang of 230 mm. It reaches the value of 0.252 for the excitation in the tangential direction and 0.205 for the excitation in the radial direction, respectively. These values are four times greater than the values obtained for larger overhangs.

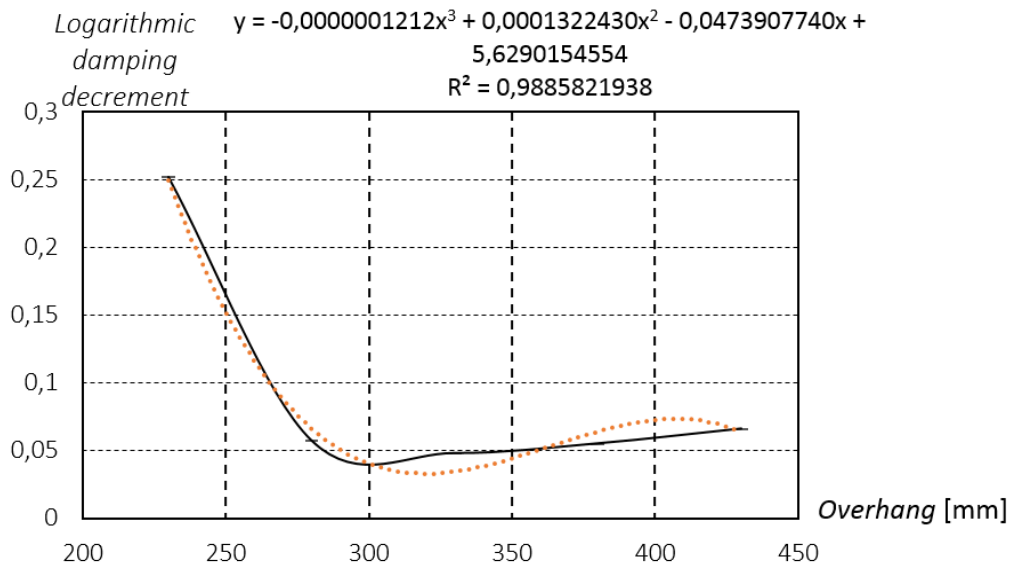


Fig 17. The course of changes in the logarithmic damping decrement value in the tangential direction for different overhangs of the mechatronic tool prototype

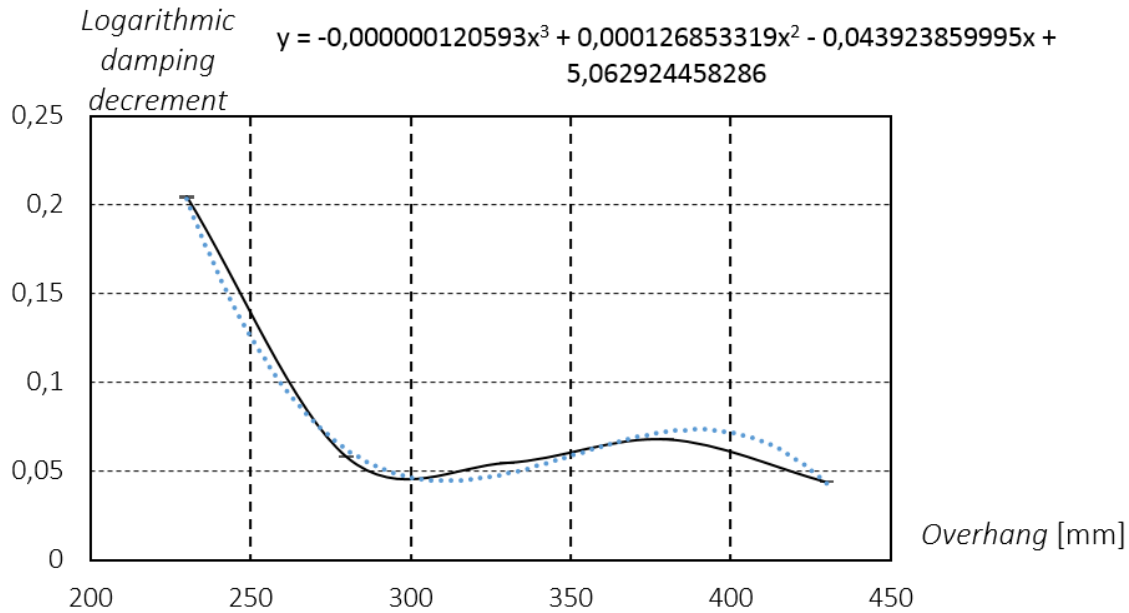


Fig 18. The course of changes in the logarithmic damping decrement value in the radial direction for different overhangs of the mechatronic tool prototype

The curves describing the variation of the logarithmic decrement of damping as a function of the variable tool attachment point are presented in Fig. 17 and 18. Their satisfactory mathematical description in terms of consistency can be realized using a third-degree polynomial function with complicated coefficients. A similar strategy for carrying out vibration measurements can be used to test other objects, for example industrial robots and vehicles, as presented by Roszkowski (2020), Skoczyński (2017), Batory and others (2024) [11, 12, 13].

Conclusion

The research results presented above allowed us to determine specific values of the tool body damping at different overhangs, as well as its resonance frequencies for variable mounting points. The tool body is characterized by a low damping value, which does not change linearly, but takes on a value approximately four times higher for the shortest overhang than the others. The nature of the changes in resonance frequencies is linear, as could be expected.

The computer data acquisition and analysis method was presented, which was successfully used to determine the spectral characteristics and the damping value of the tested mechatronic tool. Of course, an environment other than LabView™ can be used to record measurement data, it is important that it allows recording measurement data to a text file. It is also necessary to remember about the appropriately high sampling frequency of the measurement signal, minimally more than twice the expected value of the measured frequency, which results from the Kotelnikow-Shannon theorem on sampling. The range of amplitudes of the measured vibrations must also not exceed the capabilities of the used sensors.

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