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Algorithm For Establishing the Financial Profit from Reducing the Mass of the Aircraft: Optim-Based Optimization Approach*

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Abstract

In this paper, the algorithm for establishing the financial profit from aircraft's mass reduction is presented. It is applied to two examples: a passenger and a general aviation aircraft. Comparing them shows that reducing the aircraft's mass allows for high financial profit for large aircraft, while in general aviation, more revolutionary solutions are needed. Next, the OptiM software is used for determining the distance for which the revenue from taking additional passengers equals alternative financial savings from decreased fuel burn. The optimization methods applied are: Hooke-Jeeves Method, Powell's method, Steepest Descent Method, Monte Carlo method and genetic algorithms. Among those methods, the Steepest Descent method reaches the exact solution in the least number of iterations, making it the most cost-efficient method. Genetic algorithms, despite their versatility, prove to be less effective for this kind of problem.

Keywords: aerospace engineering, aviation, optimization, economy and finance, algorithm, OptiM software, MATLAB software

Introduction and Theoretical Framework

The selection, grouping and organization, storage, search and use of data is now a daily process that is encountered in almost all areas of everyday life, whether it concerns personal or professional activities. Statistical and computational problems are related to huge amounts of data that are constantly produced and for this reason there is a constant need for appropriate data processing in order to be able to draw useful and as safe as possible conclusions (Hastie, Tibshirani & Friedman, 2009). The need to improve and achieve the best possible performance of a current situation or process is a consequence of the desire for new targets that arises from the previously proposed results that resulted from an initial design. The science of mathematics and economics has contributed greatly to the methodology of the optimization process in various fields and applications that in many cases happen to be very different from each other. The principle seems to be traced to Bellman (1953) who formulated the most unified principle of Optimization by which an optimization policy has the property that whatever the initial situation and the initial decision, the following decisions must be optimal in relation to the situation that resulted from the initial decision (Mizutani & Dreyfus, 2023).

If one specifies it more, then for linear dynamic systems the optimal control policy is formulated directly in relation to the state of the system each time (see closed-loop solutions). A characteristic observation in all these cases is the resulting optimal control law calculated from the final limit to the initial one (Lazaridis, 2015). In the case of nonlinear programming for solving optimal control problems, different initial conditions, times and states

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are encountered that can be selected through the well-known Hamilton-Jacoby-Bellman (HJB) equation. This is a nonlinear first-order partial differential equation in which the generalized Hamiltonian exists and in the case where the HJB equation in question is solvable either analytically or numerically, then the corresponding optimal controllers are shown to be given by a nonlinear feedback depending on the optimized nonlinearity of the installation as well as the solution of the corresponding HJB equation (Poznyak, 2008; Sieniutycz & Jeżowski 2013).

The optimization process involves the search for extreme values of design variables, however, quite often there are multiple conflicting criteria that act competitively with each other, thus creating the need for prioritization and selection by the designer. Optimization of multiple objectives simultaneously can be assisted by the science of mathematics since it provides an appropriate mathematical framework and numerical methods for achieving the optimal design state that adapts to the various criteria required by the application. The process of systematic and simultaneous optimization of a series of objective functions is characterized as multi-objective optimization (MOO) (Stadler, 1984; Odu & Charles-Owada, 2013). At the same time, multi-criteria optimization requires simultaneous optimization of multiple, often competing or conflicting criteria (objectives). Multi-criteria decision-making (MCDM) refers to a system that helps make decisions based on multiple, but conflicting, criteria. It can also be characterized as a migrated system with a technique called multi-criteria decision analysis (MCDA). MCDA is a systematic decision-making process for selecting the most desirable and satisfactory alternative under uncertain situations (Steven, 2000). The use of MCDA is consistent with decision-making based on the decision maker's preference, which should determine which course of action or alternative would best satisfy the criteria and fully satisfy the constraints (Rao & Davin, 2008). MCDM is distinguished into Multi-Objective Decision Making (MODM) for the study of decision problems where the decision space is continuous and into Multi-Attribute Decision Making (MADM) for problems with discrete decision spaces (Zimmermann, 1991).

Methodology

Scientific research in the field of aerospace engineering allows for reducing aircraft's mass, which is a key parameter in aviation. Those innovative solutions include: new materials with better strength-to-density ratio (e.g. composites), new engines (with lower specific fuel consumption) and novel solutions in geometry (e.g. the concept of blended-wing body aircraft – Figure 1). But no matter how the mass reduction is accomplished, the question arises: how can this saved mass be utilized? In the case of passenger aircrafts, there are 3 main answers. Firstly, the aircraft can fly in its lighter version, allowing for fuel savings. Secondly, more passengers can be taken on board. And last, more cargo can be taken instead. But since cargo remains a relatively small part of airlines' revenue (around 5-10% (Walker, 2025)), the focus stays on two first solutions. Determining which option is more optimal is not straightforward, but can be preliminarily predicted using simplified models. This way, the revenue from additional passengers (further denoted as $profit_{passengers}$) and savings from less fuel consumption (further denoted as $profit_{fuel}$) can be established and compared, showing which solution is more optimal in each situation. In this paper, the algorithm which evaluates those values is introduced.

Figure 1. Advanced aircraft geometry concept – blended-wing body

Source: (Gelzer, 2010).

The conducted analysis consists of two main parts. The first part focuses on using the algorithm for two cases: flight from Warsaw to Athens and Cessna 172 Skyhawk flight. Thanks to this, the order of magnitude of financial savings achieved by mass reduction for two aircrafts of different size is shown. The second part employs the algorithm in the optimization process for establishing how the distance of flight affects which solution – taking more passengers or flying a lighter plane – is more profitable. Additionally, within this section, different optimization methods are probed.

Algorithm

To begin with, the structure of the algorithm is explained. It does not matter how the mass reduction is achieved. The value of empty weight percentage reduction is used for calculating the results.
Firstly, the algorithm for estimating $profit_{passengers}$ will be presented. This value depends on the weight per passenger, assumed to be the average weight of the European person (70.8 kg) (Walpole et al., 2012), and flight ticket price. However, the revenue cannot be calculated by multiplying this price by the number of passengers resulting from saved weight because the fees the airlines have to pay (e.g. license fees, taxes, fuel costs) decrease the revenue. In Europe, on average the airlines make around $8.20 per passenger (Spray, 2025). Since the average value is too simplified, the following formula for the revenue per passenger is proposed by the authors based on (Spray, 2025; Selim & Selim, 2018):

$$revenue\ per\ passenger = 0.16 \cdot ticket\ price + \frac{0.03}{1.5} \cdot distance\ flown \cdot fuel\ price$$

where $0.16 \cdot ticket\ price$ means that 16% of the ticket price is returned in revenue (Spray, 2025). The second element makes up for the fact that the passengers are added instead of the structure weight, so they do not make the airline pay more for fuel than for the "normal" aircraft (in which the mass is not reduced). In the above formula, the ticket and fuel price are in €, while the distance is in kilometers.

Adding passengers also increases the mass of structure. The average weight of the aircraft seat is 12 kg (Frank & Gneiger, 2017). The rest of the extra structure mass is calculated based on assumption that fuselage weighs around 10% of the maximum take-off weight (MTOW) (Kundu, 2010) and the weight of one additional row is evaluated as:

$$structure\ mass\ per\ row = 0.1 \cdot \frac{MTOW}{rows}$$

Since the number of additional rows depends on the number of additional passengers, and this value depends on the available spared mass which again depends on the number of additional rows, a loop of dependency is reached which needs to be solved in the algorithm using the *while* loop. For this paper, it is assumed that the passengers only take the hand luggage free of charge which cannot weigh more than 8 kg (LOT, 2025). Since barely ever do this luggage reaches such weight, the mass of 5 kg is assumed. Using the methodology defined above, $profit_{passengers}$ can be established.

Estimating $profit_{fuel}$ is more complex, as it requires the knowledge on mechanics of flight. The model presented here is simplified, e.g. no change of aircraft's mass through burning fuel is accounted for, but since the result is the difference between the fuel cost for the "normal" aircraft and for the lighter aircraft, the simplifications blend naturally.

To begin with, the savings from the cruise phase are considered. To understand this part of the algorithm, the four basic forces acting on the aircraft are explained (Figure 2). When the aircraft is in equilibrium, the thrust from the engine equals the drag and the weight of the aircraft is balanced by lift. From there, decreasing the weight reduces required lift. Because of the aerodynamic phenomena, lower lift generally means lower drag (Simons, 2015), so also less thrust is required, resulting in reduced fuel consumption and financial savings. In the algorithm, instead of aerodynamic lift and drag, their unitless coefficients are used, as such aerodynamic data would be used by companies.

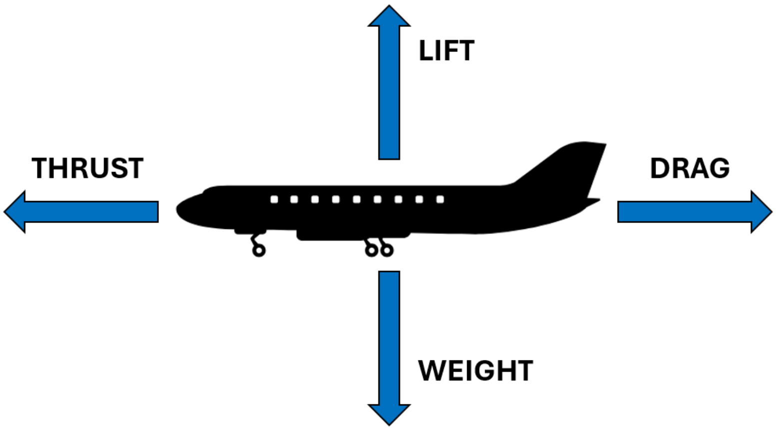


Figure 2. Basic forces acting on the aircraft

Source: Authors' own work.

Next, the distance and the cruise velocity are used for calculating the time of flight. Combining this information with specific fuel consumption gives the fuel's mass and dividing that by fuel's density results in its volume. Next, multiplying fuel price by the difference in fuel volumes for the existing aircraft and its reduced-mass counterpart gives the financial savings for cruise.

Regarding the savings for aircraft's ascending and descending, an assumption is made that during the take-off and landing, the savings are the same, so only the take-off is considered. The computation procedure is identical as for cruise. The take-off and landing phases are assumed to take 20 minutes each (Constantine, 2018).

The calculations for the two examples are conducted in MATLAB software. MATLAB is a programming and numeric computing platform suitable for various applications, both simple and complex (MATLAB, 2025). For this paper, employing this user-friendly software allowed to quickly and conveniently conduct the necessary calculations in a controlled manner.

Optimization

In the first example, a specific situation is considered, so a concrete conclusion is reached regarding which solution – adding passengers or fuel savings – is better. However, the relationship between those two is not always the same. In fact, an important parameter affecting which solution is more profitable is the distance of flight. Using the optimization process, the boundary distance can be found at which the benefit from both options is equal.

In such an approach, the distance becomes the design variable in the optimization process. The objective function, for which the minimum value is searched for, is the absolute value of the difference between the profit from additional passengers and savings from fuel:

$$objective\ function = \left|profit_{fuel} - profit_{passengers}\right|$$

The problematic part is establishing how the ticket price should change with distance, because the ticket prices are strongly affected by the day and the destination's attractiveness and this relation is ambiguous. For this paper, the same relation as before is used, with the assumption that the base ticket price is €50, which is the average for low-cost airlines (Skyscanner, 2025). Should the presented algorithm be used by airlines, the discussed relation would be known.

In this study, the following optimization methods are utilized:

- Hooke-Jeeves method – also called "pattern search", is a relatively simple, but also popular method, because it does not require calculating the derivatives (Deepgram, 2025).
- Powell's method – it is the most effective and most popular among the methods which do not use the derivatives. It is based on the concept of conjugate directions (Powell, 1964).

- Steepest Descent method – it is the first-order optimization algorithm in which local minimum is found using gradient descent by taking steps proportional to the negative of the gradient (Ahmad *et al.,* 2017).
- Monte Carlo method – contrary to the methods described above, Monte Carlo method relies on repeated random sampling, so the problems which are deterministic by nature are solved with the use of randomness (Kroese *et al.,* 2014).
- Genetic algorithms – they belong to adaptive heuristic search algorithms (Geeks for geeks, 2024), inspired by biology (concept of evolution). As in the Monte Carlo method, the element of randomness is applied, but the solutions drawn in each iteration strongly depend on the group of solutions (population) from the previous iteration.

All methods described above are implemented in the software called OptiM. OptiM is an engineering toolbox developed at the Warsaw University of Technology for numerical optimization. Optimization objective function is defined with the use of a dynamically linked library, written in C++ programming language. It can also be integrated with external programs for numerical analyses. The tools for postprocessing of the results are also available withing the software. The abovementioned optimization methods (and more) are implemented with the possibility of changing various settings influencing the convergence of the optimization process (Mieloszyk, 2022). The influence of changing these settings onto the convergence and accuracy of results has been tested in the second part of the results

The optimization methods are strongly affected by the details of how they are implemented. This especially applies to genetic algorithms. The details for OptiM software can be found in the manual in (Mieloszyk, 2017). To conclude, the results comprise three sections: 1) example 1 – flight from Warsaw to Athens, 2) example 2 – Cessna 172 Skyhawk and 3) using optimization for founding the boundary distance.

Results

Example 1 - flight from Warsaw to Athens

As first, the example of the passenger flight from Warsaw to Athens is considered. The aircraft making the flight is Boeing 737 MAX 8 (Figure 3), with its basic data summarized in Table 1.

Figure 3. Boeing 737 MAX 8

Source: (Bożyk, 2024).

Table 1. Basic data for Boeing 737 MAX 8

QUANTITY	SYMBOL	VALUE	UNIT
empty weight (structure's mass)	EW	41145	kg
maximum take-off weight	$MTOW$	82600	kg
number of rows	$rows$	30	-
number of seats in one row	$seats$	6	-

wing area	S	124.6	m^2
cruising speed	V	839	km/h
lift-to-drag ratio at cruise	K	18	-
specific fuel consumption	SFC	0.054	$kg/N/h$
velocity at take-off/landing	V_{TKOFF}	250	km/h
thrust for take-off	$T_{1\ TKOFF}$	53000	N

Source: (Boeing Commercial Airplanes, 2025; van der Zalm, 2022; Memon, 2022).

Many assumptions and simplifications result from the lack of valid data, since the airlines and aircraft producers do not share them. Nevertheless, the usefulness of the presented algorithm remains high since if the proposed algorithm was to be used in practice, the interested parties would have reliable data.

For the example flight from Warsaw to Athens, the distance is equal to $1600\ km$, and the fuel price is €1.40 per liter (dlapilota, 2022). Since the real aerodynamic data is not available, the value of the cruise lift-to-drag ratio was found and it is assumed that changing the aircraft's weight does not affect it. For the example considered, the error is not significant. The fuel's density (Jet A-1) is equal to $0.8\ \frac{kg}{l}$ (Enviro, 2021).

For ascending and descending, the computation procedure is nearly identical as for the cruise, but in the take-off and landing conditions, different values of lift and drag are achieved and no data about the lift-to-drag ratio in those phases of flight could be found. Instead, the value of thrust required for take-off (Table 1) is used for estimating the lift-to-drag ratio, again assuming that it is not changed by aircraft's weight.

Using the described algorithm, for the mass reduction of 5%, the total revenue from additional 14 passengers is equal to €470 per flight from Warsaw to Athens (assuming the ticket price of €140 (Skyscanner, 2025)). If instead the aircraft flew with reduced mass, the savings are equal to €296. So for this flight, additional passengers are more beneficial for the airlines. Even if higher passenger mass is assumed (100 kg) and the luggage of 20 kg is included in the price of the ticket, then taking more passengers is still more beneficial (€370). But for the ticket price of €53, which is offered by some airlines outside the holiday season (Skyscanner, 2025), the revenue of €277 is reached, so slightly less than $profit_{fuel}$.

Example 2 – Cessna 172 Skyhawk

For the second example, Cessna 172 Skyhawk was picked (Figure 4). This choice is not accidental, as this aircraft is world's most-produced aircraft (McCarthy, 2021). Thanks to this, its aerodynamic characteristics are available in the literature. As such, some simplifications in the algorithm can be replaced with reliable data. For this paper, data for lift and drag published in (Cel, 2014) has been used. The algorithm calculates the lift coefficient needed for flight in the same way as in example 1, but the drag coefficient is taken from the tabular data and interpolated, using the *spline* interpolation method.

Figure 4. Cessna 172 Skyhawk

Source: (Bakema, 2010).

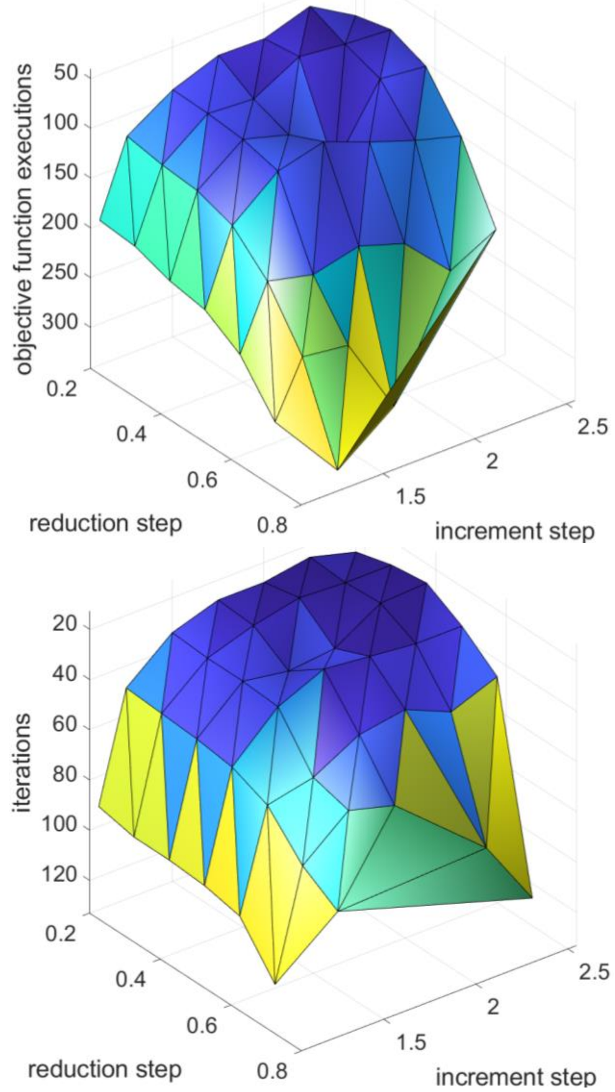
The structure's mass reduction of 5% is assumed. The data for Cessna 172 Skyhawk is as follows: $MTOW = 1157\ kg$, $EW = 754\ kg$, $SFC = 0.2534\ \frac{kg}{kWh}$, $S = 16.17\ m^2$, $V = 230\frac{km}{h}$ (Cel, 2014; Textron Aviation, 2025). Since Boeing 737 MAX 8 is a jet aircraft and Cessna 172 is equipped with a piston engine, the unit for SFC is different and that causes a slight change in the algorithm, but the general idea stays the same. The take-off (and landing) speed is $98\ \frac{km}{h}$ (Cel, 2014). The density of fuel (AVGAS) is $0.72\ \frac{kg}{l}$ (MacDonald & Peppler, 2011) and the fuel price is $2.8\frac{€}{l}$ (Rzeszow Airport, 2025). For such data, the financial savings are only $0.01\ \frac{€}{km}$. Typically, the flight of this aircraft does not take more than an hour, resulting in savings of only €2.30 per flight. It can be seen that employing new technologies in mass reduction would allow for financial savings mostly for the large aircraft, while general aviation needs more revolutionary solutions to reach significant changes.

Optimization - boundary distance

In the optimization process, the objective function is the absolute value of the difference between the profit from additional passengers and savings from the lighter aircraft, while the design variable is the distance flown (range). In the calculations, various optimization methods are tested with different settings, distinct depending on the method.

Firstly, the Hooke-Jeeves method (HJM) is considered. No matter the settings, the objective function reaches 0, so the exact solution is found. The boundary distance for which profits from both solutions are the same equals $758.51\ km$. This value applies to the assumptions and values of the parameters introduced in example 1. If another situation was considered, e.g. higher ticket prices or lower mass of the aircraft, different boundary distance would be reached.

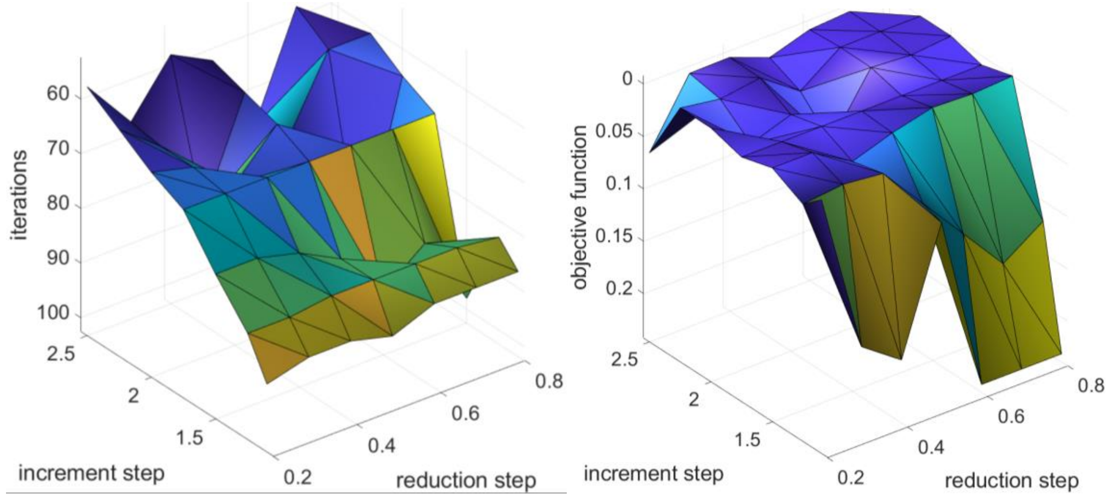
For HJM, two main parameters can be changed in the software: increment step and reduction step. Figure 5 shows how changing these values affects the number of iterations (NoI) for convergence and how many times the objective function is executed ($F_{obj\ exe}$). Some differences between both plots appear, showing that sometimes changing the setting can lead to less iterations, but the objective function needs to be called more times, leading to higher calculation time. For faster convergence, it is beneficial to increase the increment step and reduce the reduction step, although not indefinitely. The fastest convergence is reached for increment step of 2.25 and reduction step of 0.2. The influence of changing the reduction step is more significant than the increment step. Figure 5 does not show all generated data, as NoI and $F_{obj\ exe}$ increase significantly for low increment step and high reduction step and those cases hinder the plots' readability.



(a) objective function executions (b) number of iterations

Figure 5. Influence of Hooke-Jeeves method parameters on convergence.
Source: Authors' own work.

Secondly, the Powell's method (PM) is considered. Here, the relation between NoI and $F_{obj\ exe}$ is unambiguous, so only the former is shown in Figure 6a. However, contrary to HJM, the objective function does not always reach 0 (Figure 6b). Nevertheless, the difference is not large compared to the order of magnitude. For small increment step, the error does not appear. The relation between the parameters of the method (increment step and reduction step) is inconclusive compared to HJM, but in general increasing increment step improves convergence. Changing the reduction step barely influences NoI for small increment step. For higher increment step, the significance of the reduction step is high, but its relation with NoI becomes highly irregular and nonlinear.



(a) number of iterations (b) objective function value

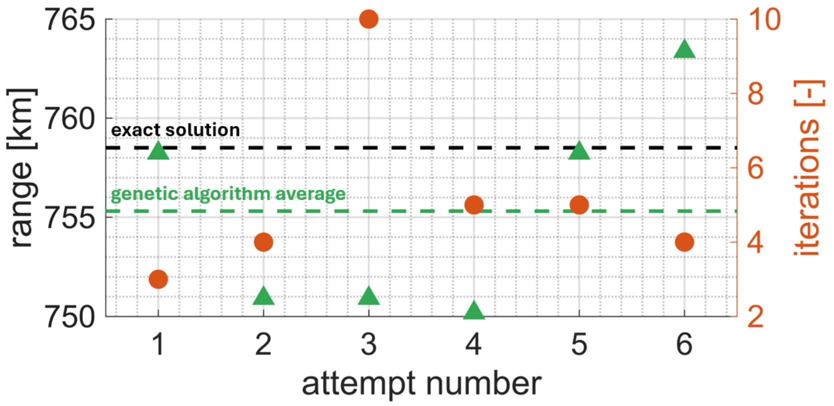
Figure 6. Influence of Powell's method parameters on convergence and accuracy of results
Source: Authors' own work.

Next, the Steepest Descent method (SDM) is applied to the problem. For adjusting the length of the step in each iteration, the Armijo condition is used (Asl & Overton, 2018). The method achieves the exact solution to the problem in just 7 iterations. Thanks to adjusting the settings of the Armijo condition, $F_{obj\ exe}$ could be reduced – from 54 for linear interpolation to 42 for quadratic interpolation. Surprisingly, using single-sided derivatives requires less iterations than double-sided derivatives and better accuracy is achieved.

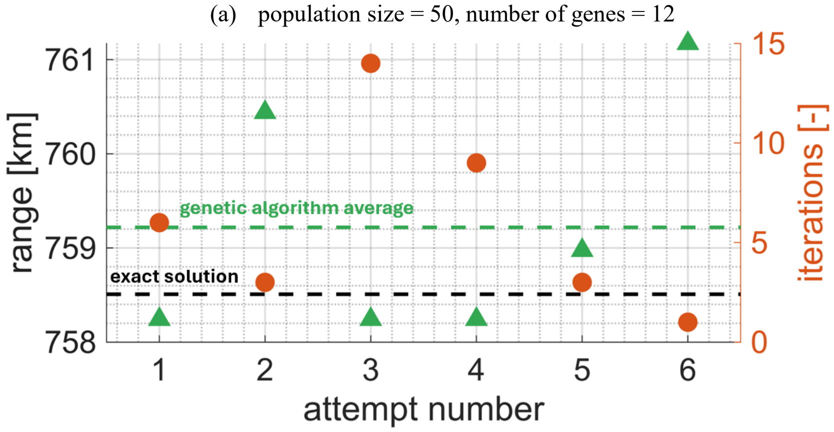
As for the Monte Carlo method (MCM), despite its randomness, the exact solution is found for most cases. The number of samples can be decreased to up to 25 (from default value of 100) for reaching satisfactory results. Decreasing the resolution parameter significantly reduces the calculations time while the accuracy remains nearly perfect. The radius parameter can be decreased from its default value of 0.5 to 0.2. This way, NoI decreases from 16 to 7 without losing accuracy. Further reducing this parameter causes significant divergence from the exact solution.

As the last method, a genetic algorithm (GA) is used. For the strategy of selecting the parents, the Roulette method is applied. Tests for the Tournament method were also conducted, but no significant differences between those two approaches have been observed. Two main parameters of the method are changed: the population size (the number of solutions for each iteration) and the number of genes (which affects the achievable resolution). Since the computations for GA are strongly affected by randomness, for each considered set of parameters, six attempts were conducted. Each attempt returned different results in different NoI, as shown in Figure 7. Green points show the result (range) reached in each attempt and orange points correspond to NoI.

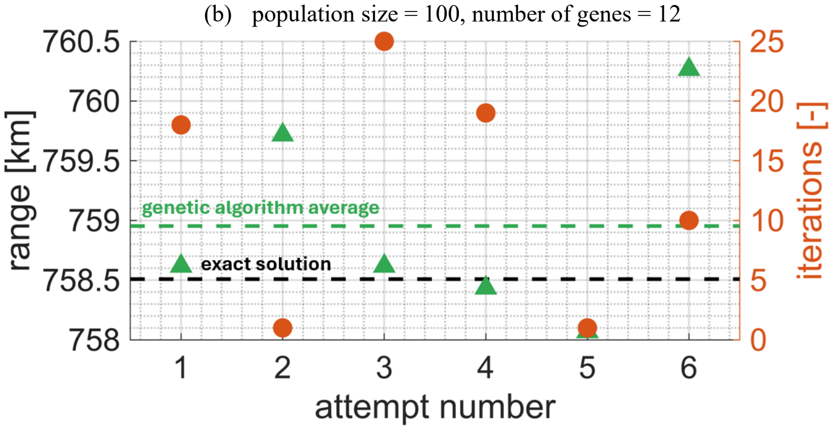
$F_{obj\ exe}$ is not shown in Figure 7, but it can be calculated by multiplying NoI (plus one, as the iteration number zero is choosing the first population) by the population size. From there, it can be concluded that the computational cost for GA is significantly higher than for e.g. HJM or SDM, but for cases in Figure 7a-c, it is lower than for PM.



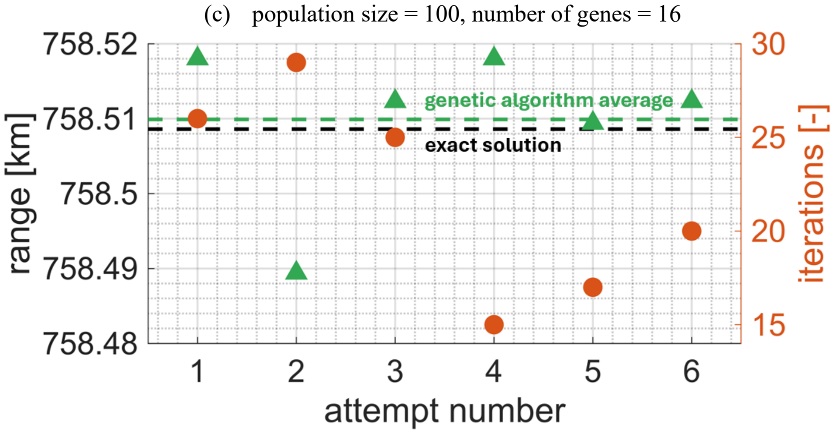
(a) population size = 50, number of genes = 12



(b) population size = 100, number of genes = 12



(c) population size = 100, number of genes = 16



(d) population size = 100, number of genes = 20

Figure 7. Influence of genetic algorithm's parameters on convergence and accuracy of results

Source: Authors' own work.

Both increasing the population size and the number of genes leads to significant improvement in accuracy, lowering the dispersion between attempts. However, NoI also increases, which also increases the computation time. Apart from individual solutions for each attempt, Figure 7 also shows the average value of the boundary distance (range). Increasing the population size from 50 to 100 decreases the divergence of the averaged result from the exact solution from 4 to 1 km (Figure 7a-b). Increasing the number of genes from 12 to 16 further reduces this difference to 0.5 km (Figure 7c) and for the most computationally costly case (population size = 100, number of genes = 20), the exact solution is reached with minimal error (Figure 7d).

Summing up, in terms of accuracy, two of the presented methods found the exact solution no matter the settings, i.e. HJM and SDM. Regarding the computational cost, although the MCM takes more objective function executions than some of the other methods, the time needed for calculations is the lowest if the resolution parameter is reduced. Second fastest method is SDM with $F_{obj\,exe} = 42$ in 7 iterations, third is PM with $F_{obj\,exe} = 57$ for large increment step and small reduction step and the next one is HJM with $F_{obj\,exe} = 62$ in similar conditions.

Summary and conclusions

In this paper, the algorithm for establishing the financial profit from reducing the mass of the aircraft has been presented. Two ways of utilizing this reduction of mass are shown: taking more passengers or flying a lighter aircraft. Next, the algorithm is applied to two examples: a passenger aircraft flying from Warsaw to Athens and a general aviation aircraft. In the last part of the analysis, the optimization process using the presented algorithm is conducted for determining the boundary distance (range) for which the revenue from taking more passengers equals the financial savings from flying the lighter aircraft. Various optimization methods were applied and the susceptibility of the solution to the settings of those methods is explored.

Based on the presented results, the following conclusions can be drawn:

- reducing the mass of the aircraft allows for high financial profit for large, passenger aircraft, while in the field of general aviation, more revolutionary solutions are needed,
- among presented optimization methods, the Steepest Descent method reaches the exact solution in the least number of iterations, making it the most cost-efficient method for solving this kind of problem without losing the accuracy,
- Hooke-Jeeves method proves to be somewhat more appropriate than Powell's method for the considered problem with significantly lower number of objective function executions needed for convergence. Both methods' convergence is strongly affected by setting the proper size of the increment and reduction step,
- Monte Carlo method copes well with the problem even for relatively small number of samples. Other parameters could also be reduced from their default values, speeding up the optimization process without losing the necessary accuracy,
- genetic algorithms are appropriate for solving problems with multiple local optima which hinder simpler methods from finding the global minimum. However, in the case of a relatively simple problem like the one presented in the paper, the computational cost of this method is significantly higher than for the more basic methods (e.g. Hooke-Jeeves method) while the accuracy is worse.

The conducted analysis is burdened with some limitations. First of all, the model of the aircraft is highly simplified, e.g. the mass is assumed to not change during the flight. But because the consumption of fuel is calculated based on the same assumptions for two cases and then compared, the error is partially reduced. Next, some of the numbers had to be arbitrarily chosen because the companies do not share their data. This also lead to further assumptions which simplify the problem. However, for the example of Cessna 172 Skyhawk, the aerodynamic data is available and it turns out that the assumptions made for flight from Warsaw to Athens can be deemed correct (e.g., the cruise lift-to-drag ratio was assumed to be constant in the first example and applying real aerodynamic data in the second example shows that the change is not significant).

As for further work, alternative ways of modelling different aspects of the aircraft should be applied to check whether significantly distinct results are achieved. For instance, the formula for mass of structure per one additional row can be replaced with more traditional formula of how MTOW is affected by changing mass of payload. This formula utilizes typical shares of fuel mass in total mass and empty aircraft mass in total mass for a given type of aircraft (Raymer, 2012). Some aspects of flight which were completely omitted in the algorithm proposed in this paper can be developed to increase the accuracy. Also, more examples than only extreme cases should be considered. As for optimization, the influence of parameters other than range on what is the optimal technique of utilizing reduced mass should be explored and more thorough sensitivity analysis is also needed. This way, more detailed information regarding potential financial profits from reducing the mass of the aircraft can be gathered. Such information would help airlines in determining what is the optimal strategy they should use in various situations, depending on details of the particular flight, airline's fleet, etc., leading to better decision-making.

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