IBIMA Publishing
Communications of International Proceedings
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Vol. 2025 (20), Article ID 4535325

Simulation of Determination of Positioning Errors Of A Robotic Manipulator*

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* Presented at the 45th IBIMA International Conference, 25-26 June 2025, Cordoba, Spain

Abstract

Credible assessment of positioning errors of industrial robots is important for their application. The paper concerns a problem of determination of positioning errors of a manipulating robot basing on simulated experimental results. The procedure of ISO 9283 standard was the base of determination of 5 measurement points. A suggested set containing 9 points was also used. Kinematic DH model of a manipulator was applied to determine the measurement points. Actual poses were obtained by introduction of small arbitrarily selected changes to parameters of the nominal kinematic model. Simulated measurement results were obtained by modification of actual coordinates by adding pseudorandom errors. Determination of the positioning error was preceded by alignment of the robot base frame to the instrument coordinate frame. Next, the positioning errors were calculated with respect to the aligned base frame. An attempt to improve the initial results consisted in an iterative updating of the alignment using results of zero joint coordinates offsets estimation. In the result of the simulation, the average error of estimation of the positioning error in 9 points was reduced from about 65% to about 7%. The simulation results confirmed that erroneous alignment of the robot base frame to the instrument frame significantly influenced accuracy of determination of the positioning error, and that this accuracy might be improved by iterative updating of the joint coordinates values with the estimated zero joint offsets.

Keywords: industrial robots, robotic manipulator, robot positioning errors

Introduction

General purpose computing environments, like Matlab used in the reported research, according to Mathworks Inc. webpage (2025), are very useful for solving technical problems. By providing possibility of: efficient numerical calculation, symbolic derivation, and programing, as well as a comprehensive graphical presentation of results, such the environments considerably ease and speed up investigations.

Precision of positioning of manipulating robots used in industrial manufacturing should be assessed, according to ISO 9283 (1998) standard, by positioning repeatability (RP) and positioning accuracy (AP). Usually, only appropriate repeatability is demanded. For some operations containing interaction of robot's end-effector with objects or collaboration of a robot with other positioning mechanisms, achieving satisfactory positioning accuracy may be also required. ISO 9283 (1998) standard stated that positioning accuracy should be determined experimentally basing on estimation of positioning errors with respect to the robot's base frame. For that purpose, measurement of position is needed. In the procedure of measurement of position, the most often, laser trackers, like reported by Wu et al (2025), or vision systems, like in the research of Svaco et al (2014), are used. Wang et al (2015) stressed that such the measurement is usually carried out with respect to a measuring instrument's coordinate frame. Thus, like mentioned by Franaszek and Marvel (2022), determination of the positioning errors requires finding a relation between the robot base frame and the instrument coordinate system. This could be caried out using parameter estimation procedure based on the manipulator's kinematic model and results of measurement of a set of the manipulator's end-effector positions.

Cite this Article as: Wojciech LISOWSKI, Vol. 2025 (20) "Simulation of Determination of Positioning Errors of a Robotic Manipulator" Communications of International Proceedings, Vol. 2025 (20), Article ID 4535325, https://doi.org/10.5171/2025.4535325

Robot controllers, during positioning of the manipulator's end-effector, apply the nominal manipulator's kinematic model, which has values of parameters specified by the robot manufacturers. Differences between these nominal values and the actual values constitute a significant, but not the only one, cause of positioning errors. This motivated engineers for developing of robot calibration methods like reported in Roth et al (1987), Mooring et al (1991), as well as in Bernhard and Albright (1993). Such the methods comprise experiment-based parameter identification followed by positioning errors' correction procedures. Robot calibration, which is being continuously improved, proved to be effective in minimizing positioning errors of manipulating robots. Examples are reported in Svaco et al (2014), Wang et al (2015), and in Cheng et al (2024). Parameter identification methods, used in the course of the robot calibration procedures, can be applied also for improving quality of estimation of a robot end-effector's positioning error.

This paper concerns problem of determination of positioning errors of an end-effector of a robotic manipulator. Basing on the simulation results, the most important problems of determination of positioning errors were to be identified and effectiveness of the formulated algorithms of analysis to be verified. The reported investigation was undertaken as a preparation stage to the planned laboratory tests.

Procedure of investigation

Determination of AP, according to ISO 9283 (1998), consists in experiment results-based determination of positioning errors of the robot end-effector. AP relates the command pose to the position determined by averaging coordinates of attained poses in a series of approaches to the command pose. Positioning error, being the distance between the command (nominal) pose and the attained (actual) pose, characterizes each positioning action. As in reality, the actual pose is unknown, usually, the position error is approximated by the distance between the measured position transferred to the estimated robot base frame and the nominal position, like in papers of Wang et al (2015), and Franaszek and Marvel (2022). The aim of the reported research was to attempt to improve the initial results of determination of positioning errors for 5 points selected according to ISO 9283 (1998) recommendations by introduction of iterative updating of the alignment between the instrument frame and the robot base frame using results of estimation of the robot zero joint coordinates offsets. For comparison purpose, increased to 9, number of measurement points was also considered aiming at improvement of results of the carried-out parameter estimation.

Geometrical modelling of robotic manipulators commonly is carried-out using Denavit-Hartenberg (DH) model reported in a book of Ranky and Ho (1995). It applies homogeneous transformation matrix for representation of position and orientation of *i*-th coordinate frame with respect coordinate frame No. *i*-1, which are assigned to the considered manipulator's rigid links. The structure of DH model is presented by formula (2.1).

$$^{i-1}\mathrm{T}_i = Rot(z_{i-1},\theta_i)Tra(z_{i-1},d_i)Tra(x_{i-1},a_i)Rot(x_i,\alpha_i) \qquad (2.1)$$

where: *Rot* - states for rotation, *Tra* - for translation, *x* and *z* are axes of the considered coordinate frames, θ and α - are rotation angles, and *d* and *a* – are translational parameters.

For further analysis the elements of T matrix were labeled in the following way (2.2):

$$\mathrm{T} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_1 \\ R_{21} & R_{22} & R_{23} & P_2 \\ R_{31} & R_{32} & R_{33} & P_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.2)$$

For a six rotary axis of motion manipulator (6R), which is considered in this research, parameters: θ, d, a, and α are interpreted as: the joint variable, the link offset, the link length, and the link twist angle, respectively. DH model is the most popular one and practically useful, because it is implemented in a vast majority of industrial manipulating robot controllers. The structure of the model is appropriate for planning motion of a robotic manipulator in space, but it is not sufficient for a precise representation of a general geometrical error of dimensions and/or mutual alignment of a robotic manipulator components. In the reported simulation, the positioning error was assumed to be composed of a deterministic and a random part. The deterministic component was modelled by the arbitrarily chosen deviations of 4 parameters of DH model (2.1). First, by the dominating, like mentioned by Lee et al (2017) and Franaszek and Marvel (2022), zero joint (variable/coordinate) offset $\delta\theta$, then, by the remaining: δd, δa, and $\delta\alpha$ errors. The random component was modelled by a pseudorandom deviation of coordinates of position from the nominal values, and it was introduced to account for both the positioning repeatability errors and the position measurement errors. The measured positions X_M of the selected point of the robot end-effector, referred to as the Tool Centre Point (TCP), must be transformed to the robot base frame using $^{B}T_{I}$ homogeneous transformation matrix. The measured with respect to the base frame positions X_m are determined according to formula (2.3). Matrix $^{B}T_{I}$ can be estimated basing on the simulated measurement results. For estimation

Matlab's *lsqnonlin* algorithm was used like stated in Mathworks Inc. webpage (2025). The minimized error equations had the form (2.4).

$$\mathrm{X}_m = {}^{B}\mathrm{T}_I \mathrm{X}_M \qquad (2.3)$$

$$\mathrm{e} = \mathrm{X}_m - {}^{B}\mathrm{T}_{IE}\mathrm{X}_M = \begin{bmatrix} x_{m1} \\ x_{m2} \\ x_{m3} \\ 1 \end{bmatrix} - \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_1 \\ R_{21} & R_{22} & R_{23} & P_2 \\ R_{31} & R_{32} & R_{33} & P_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{M1} \\ x_{M2} \\ x_{M3} \\ 1 \end{bmatrix} \qquad (2.4)$$

Initially, X_m was unknown and nominal positions X_n were used instead. In the result of the estimation procedure 12 elements of ${}^{B}T_{IE}$ matrix were determined. For the case of 5 measurement points, the number of the error equations was found to be insufficient, and parameters R_{13}, R_{23}, and R_{33} were determined separately taking into account that vector $[R_{13},R_{23},R_{33}]^T$ is a vector product of: $[R_{11},R_{21},R_{31}]^T$ and $[R_{12},R_{22},R_{32}]^T$ vectors, taking advantage of orthogonality of these vectors mentioned by Ranky nad Ho (1985).
Zero joint offsets $\delta\theta$ were estimated in the same way, taking into account the relationship between X_n and joint variables θ (2.1), constituting the forward kinematics of manipulators discussed in Ranky and Ho (1985), and substituting the simulated nominal joint variables q_n. The minimized error equations had the form (2.5).

$$\mathrm{e} = \mathrm{X}_n(q_{ni} + \delta\theta_i) - {}^{B}\mathrm{T}_{IE}\mathrm{X}_M \qquad (2.5)$$

The formulated simulation procedure aimed at determination of the positioning errors based on the simulated values of the joint variables q_n, the nominal DH model of the manipulator, and simulated measured positions X_M of TCP with respect to the instrument coordinate frame. The formulated and applied algorithm of simulation is presented in Fig. 2.1.

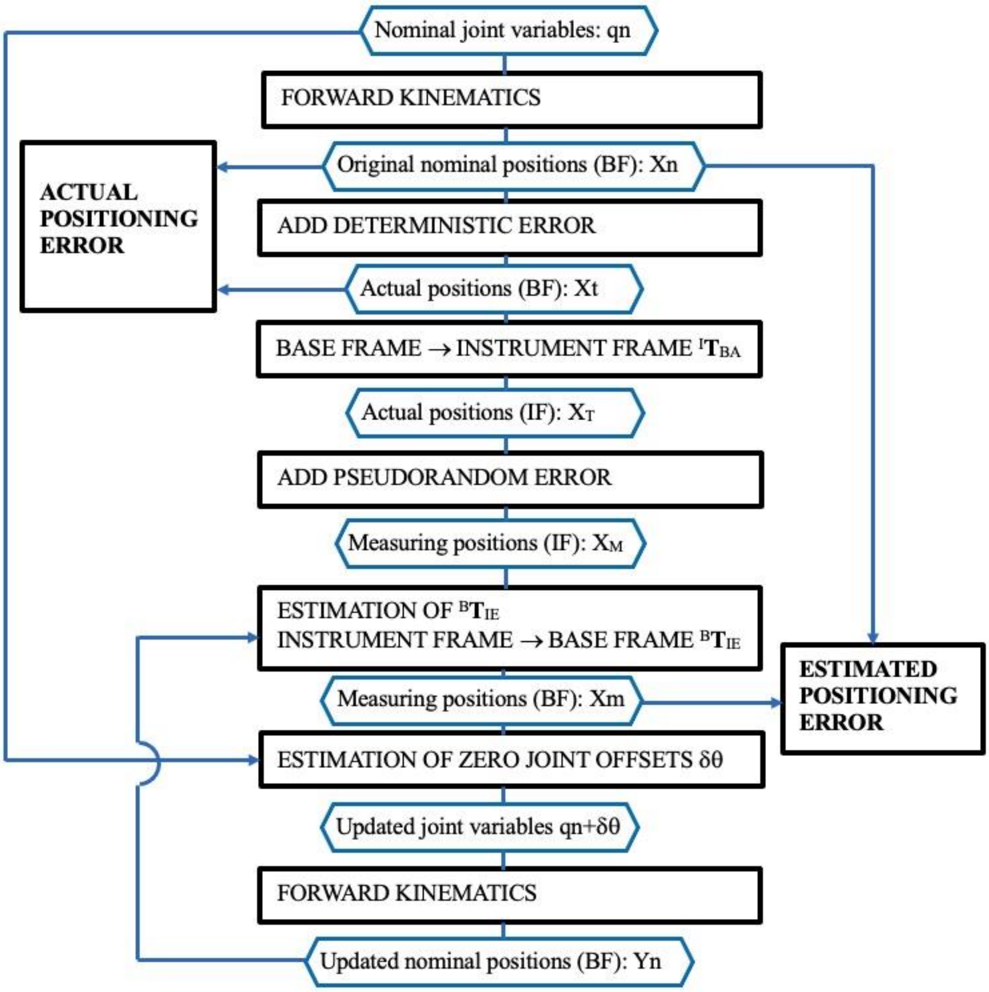

Fig. 2.1. Algorithm of the conducted simulation of determination of positioning errors

First, values of the nominal joint variables q_n were used to determine the nominal positions X_n of TCP with respect to the robot base frame. The chosen values of the deterministic errors were added to the parameters of the nominal DH model, which was used for calculation of the actual positions X_t. Next, the position and orientation of the frame assigned to the

measuring instrument relative to the robot base frame was assumed. It was represented by the homogeneous transformation ${}^{B}T_{IA}$ matrix. Its inverse was used to determine actual positions X_T of TCP with respect to the instrument coordinate system. Simulation of the measurement of TCP position consisted in adding a pseudorandom correction to X_T 30 times to data representing each of the measurement points. This resulted in creating of 150 or 270 measurement results for 5 or 9 measurement points, respectively. This measurement results were averaged, like in ISO 9283 (1998), separately for each of the measurement points before the further analysis. During the first parameter estimation procedure, 12 values of elements of ${}^{B}T_{IE}$ matrix were found. This matrix was used to transform the measured averaged positions X_M to the estimated base frame. Values of the measured positions expressed in the estimated robot base frame X_m and the nominal values of the joint variables q_n, were used, in the second parameter estimation procedure, for determination of the estimates of the zero joint offsets $\delta\theta$. Basing on these values, the nominal values of the joint variables q_n were updated, and taking advantage of the nominal DH model, used in determination of the updated nominal positions Y_n relative to the robot base frame. This initiated the iterative 2 stage parameter estimation procedure, in which the updated values of nominal joint variables q_n, of the positions Y_n, and the of the measured positions X_M were used to improve estimation of the positioning error, calculated as the distance between X_n and updated X_m. The actual positioning error, which is unknown in reality, for the purpose of assessment of quality of estimation of the positioning error, was determined basing on the distance between the nominal positions X_n and the actual positions X_t of the measurement points.
In the course of the carried-out simulation the following assumptions were made:

- a manipulator of 6 rotational motion axes (the anthropomorphic arm with attached spherical wrist) was considered
- parameters of the simulated manipulator corresponded to KUKA KR 6 Agilus sixx R900 robot according to KUKA KR 6 Operating Instruction (2015)
- only parameters used in DH model were taken into account
- simulated measurement results were referred to the measuring instrument coordinate frame
- approximated pose of the robot's base frame was determined with respect to the measuring instrument coordinate frame
- direction of approach to the command position during the experiment was not modelled and its influence on positioning errors was neglected.

In the next section, the detail description of the input data for the conducted simulation were reported.

Simulation set-up

For the purpose of the simulation, sets of: nominal positions X_n, actual positions X_t of the measurement points, simulated measuring positions X_M were created using algorithms discussed in the following subsections.

Nominal positions of the measurement points

Determination of positions of the manipulator's end-effector requires formulation of the nominal kinematic model consistent with Denavit-Hartenberg theory reported in Ranky and Ho (1985). A kinematic diagram of the considered manipulator with assumed coordinate frames is presented in Fig. 3.1. Coordinate frame No. 7 with an origin located at the position of the measurement point (TCP) was added to the diagram of the manipulator of the 6R kinematic structure. Parameters of the nominal kinematic model are listed in Tab. 3.1.

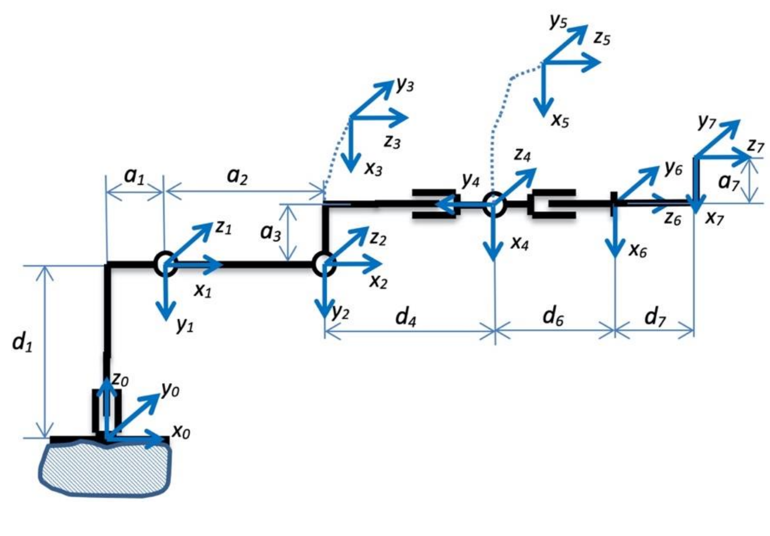

Fig. 3.1. Kinematic diagram of a manipulator with assumed coordinate frames

Tab. 3.1. Parameters of nominal kinematic model (DH) of the considered manipulator

Link No.	θ	d	a	α
1	θ_1	d_1	a_1	$-90°$
2	θ_2	0	a_2	$0°$
3	$\theta_3+90°$	0	a_3	$90°$
4	θ_4	d_4	0	$-90°$
5	θ_5	0	0	$90°$
6	θ_6	d_6	0	$0°$
7-TCP	$0°$	d_7	a_7	$0°$

The following values of the parameters indicated in Fig. 3.1 and Tab. 3.1 were assumed (in meters): d_1=0.4, a_1=0.025, a_2=0.455, a_3=-0.035, d_4=0.42, d_6=0.08 according to KUKA KR 6 Operating Instruction (2015). Values of d_7=0.08, and a_7=-0.05 corresponded to the element available at the author's laboratory, which was to be used in the planned experiment.

The procedure of determination measurement points' (P1 to P5) position according to ISO 9283 (1998) was based on placement of a cube of the possibly maximum volume inside the tested manipulator arm's workspace, in its part corresponding to typical operations of the robot. The envelope of this workspace is a set of extremal locations of the manipulator's 3rd link tip, being: a position of the wrist motion axes intersection and a location of the origin of the 4th local coordinate frame O_4 (Fig. 3.2). The cube possesses 8 vertices (from C1 to C8). It was assumed that vertices C1, C2, C5 and C6 are located on the envelope of the manipulator 3rd link tip's workspace. The formulated algorithm of determination of C1 vertex, illustrated in Fig. 3.2, is starting from point S, which is a location of O_4 for θ_1=0, for links No. 2 and 3 horizontally extended along axis x_1, parallel to x_0 axis of the manipulator base frame, passing though the centre of the 1st local coordinate frame O_1. Location of C1 is given by: x_1, l_y and d_1+l_z coordinates. The calculations are conducted for a set of gradually increasing values of l_x distance. The value of R_v is a radius of the robot arm's workspace envelope in a vertical plane, and it can be calculated according to formula (3.1).

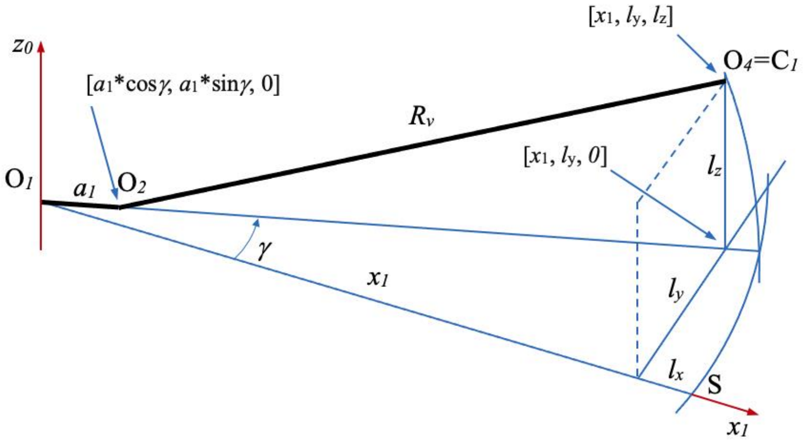

Fig. 3.2. Illustration of parameters of the algorithm of determination of position of the cube's vertex C_1

$$R_v = a_2 + \sqrt{a_3^2 + d_4^2} \qquad (3.1)$$

Parameter R_h (3.2) is a radius of the robot arm's workspace envelope in the horizontal (x_0y_0) plane.

$$R_h = |\overline{O_1S}| = a_1 + R_v = x_1 + l_x \quad (3.2)$$

For a selected value of l_x, values of l_y and l_z, which are to be equal each other are determined. Radius R_v is a distance between O_2 and O_4 points, and as such, it satisfies condition (3.3). Next, taking into account that angle γ can be found using formula (3.4), a value of $l= l_y= l_z$ is determined.

$$R_v^2 = (x_1 - a_1 * cos\gamma)^2 + \left(l_y - a_1 * sin\gamma\right)^2 + (l_z - 0)^2 \qquad (3.3)$$

$$tan\gamma = \frac{l_y}{x_1} \qquad (3.4)$$

The coordinates of C1-C8 points were determined according to Tab. 3.2. In the consequent iterations, following the increase of l_x distance value, the cube's volume was also rising. The calculation was carried out until an assumed stop condition was met. In the considered case two conditions were applied. The first one was the limitation of the vertical motion of the manipulator's end-effector by the station's table (approximately located at z=0). The second condition was corresponding to the distance between C1 and C7 (or C2 and C8), being a length of the cube's diagonal, which must not be greater than the measuring range of the instrument planned to be used.

Tab. 3.2. Coordinates of C1-C8 points ($l=l_y=l_z$)

	C1	C2	C3	C4	C5	C6	C7	C8
x	x_1	x_1	x_1-2l	x_1-2l	x_1	x_1	x_1-2l	x_1-2l
y	l	-l	-l	l	l	-l	-l	l
z	d_1+l	d_1+l	d_1+l	d_1+l	d_1-l	d_1-l	d_1-l	d_1-l

The conducted calculations were finished after the first mentioned stop condition was satisfied. Fig. 3.3 presents variation of location and size of the cube's diagonal plane during calculations. The rectangle of the greatest circumference length indicated the selected positions of vertices: C1, C2, C7, and C8. Next, positions of TCP corresponding to C1, C2, C7, and C8 were determined according to ISO 9283 (1998). The first step of this procedure consisted in choosing the first point at the centre of the rectangle C1C2C7C8. The positions of the remaining 4 points were determined by shifting C1, C2, C7, and C8 towards the centre by the distance of 10% of the rectangle's diagonal length (e.g., C1C7 segment). The orientation of the tool required by the standard, using RPY angles presented in Ranky and Ho (1985), was set to [180°,45°,180°]. Taking it into account and using kinematic diagram of the manipulator (Fig. 3.1), the achieved positions were translated, next, by a vector given by formula (3.5).

$$[d_6 cos(45°) \quad 0 \quad -d_6 sin(45°)]^T \qquad (3.5)$$

This transformation was needed to determine position of the wrist-flange centre corresponding to them. The determined coordinates were rounded to the full mm, which later should help to avoid introduction of rounding errors during programing the real robot motion.

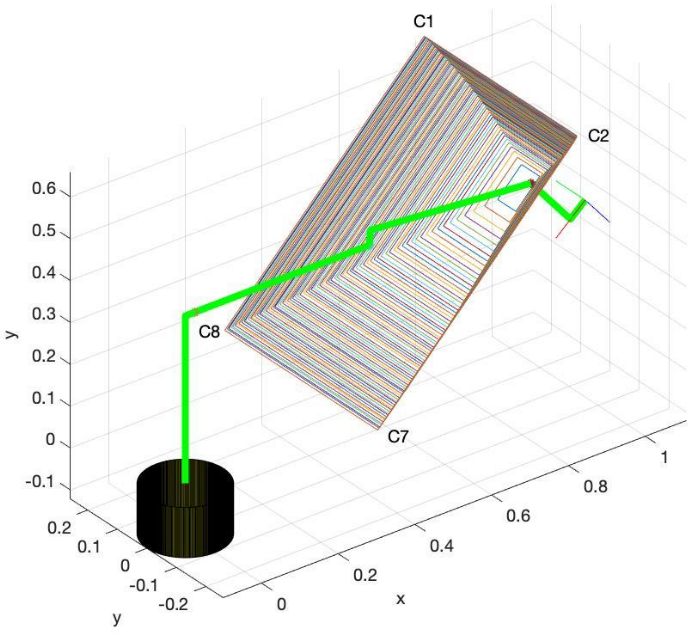

Fig. 3.3 Variation of location and size of the cube's diagonal plane in the course of calculation

Finally, the rounded, with keeping shape of the cube, coordinates were again shifted by adding the vector defined by formula (3.6) accounting for geometry of the end-effector.

$$[(d_7 - a_7)cos(45°) \quad 0 \quad (-d_7 - a_7)sin(45°)]^T \qquad (3.6)$$

The mentioned additional 4 measurement points (P6, P7, P8 and P9) were created in the corresponding way basing on locations of C3, C4, C5, and C6 vertices. The distribution in space of the measurement points is presented in Fig. 3.4.

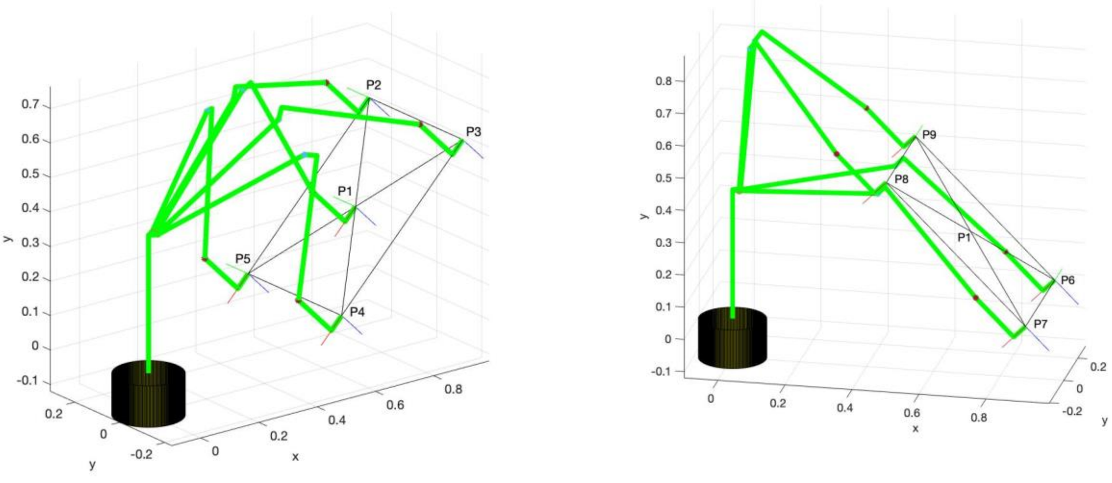

Fig. 3.4. Nominal positions of the measurement points.

The nominal coordinates of the determined measurement points were rounded to 0.01 mm.

Actual positions of the measurement points

Creation of the actual positions X_t of TCP basing on the nominal ones X_n requires making assumption about the values of the positioning errors, which would provide realistic and substantial results of the simulation. The first considered indicator was a range of values of the positioning errors reported in references: Tovar-Ariaga et al (2012), Svaco et al (2014), Wang et al (2015), Cheng et al (2024), between 1 and 6 mm. This range was chosen as the reference for assessment of the simulation set-up. Next, values of variation of parameters of the DH model (2.1) were to be set. For the formulated DH model deviations of 28 parameters from their nominal values could be considered. As all the measured positions were to be determined with respect to the instrument frame, separate estimation of zero joint offset $\delta\theta_1$ from estimation of the alignment transformation ${}^{B}T_{IE}$, would require additional experiment. Thus, $\delta\theta_1$ was set to 0. The remaining values of zero joint offset ($\delta\theta_i$ i=2, …,6) were set to 0.5° basing on a value corresponding to the very coarse general tolerance for angular dimensions as stated in ISO 2768 (1989), basing on an assumption that they correspond mainly to assembly. The linear parameters δd and δa correspond to manufacturing, and their values were assumed to be 0.04 mm for IT6 tolerance grade for dimension over 400 mm according to ISO 286-1 (2010). Similarly, values of $\delta\alpha$ were set to 0.083° basing on ISO 2768 (1989) for fine general tolerance for angular dimensions. All the assumed values of errors of the geometrical parameter are listed in Tab. 3.3. The signs of the errors were selected arbitrarily, so that $\delta\theta_2$ and $\delta\theta_3$ had opposite signs. Assuming of the same sign of the mentioned errors resulted in about 2 times greater values of the positioning error. The obtained values of the positioning errors calculated for the selected values of the assumed errors of the parameters of the formulated DH model are listed in Tab. 3.4.

Tab. 3.3. Assumed values of errors of geometrical parameters (angles in degrees, lengths in mm)

i	1	2	3	4	5	6	7
$\delta\theta_i$	0	0.5	-0.5	0.5	0.5	-0.5	0
δd_i	0.04	-0.04	0.04	-0.04	0.04	-0.04	0.04
δa_i	0.04	-0.04	0.04	-0.04	-0.04	0.04	-0.04
$\delta\alpha_i$	0.083	0.083	-0.083	0.083	-0.083	0.083	0

Tab. 3.4. Actual deterministic positioning error (mm)

Points:	P1	P2	P3	P4	P5	P6	P7	P8	P9
Only δd δa $\delta\alpha$	1.71	1.26	1.26	2.00	1.84	1.79	1.68	1.25	1.31
with $\delta\theta$ added	4.67	5.22	4.74	4.80	5.15	4.20	4.49	4.29	3.37

These values were found to fit the assumed range of values reported in the considered references

Simulated measurement results

The results of the simulated measurements were expressed with respect to the instrument coordinate system, which was assumed to possess position and orientation relative to the robot base frame given by matrix $^{B}T_{IA}$ (3.7).

$$^{B}\mathbf{T}_{IA} = \begin{bmatrix} -0.866004 & 0.499962 & -0.008664 & 0.750 \\ -0.499988 & -0.866032 & 0.001044 & 0.550 \\ -0.006981 & 0.005236 & 0.999962 & 0.001 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.7)$$

After determination of this matrix, the actual positions X_t were transferred to the instrument frame. Next, the magnitude *r* of the deviation of the measured positions X_M relative to the actual ones X_T resulting from unmodelled phenomena was determined using Matlab pseudorandom numbers generator *randn* (0.015 mm mean and 0.01 mm standard deviation) accordingly to description on Mathworks Inc. webpage (2025). These values were assumed to correspond to the approximated combined error of positioning repeatability of the considered manipulator and the instrument measurement error. For each measuring point 30 measurements were simulated. Coordinates of each positioning error (h_x, h_y, h_z) were calculated for each simulated measurement according to formula (3.8).

$$\begin{aligned} h_x &= r cos\theta sin\varphi \\ h_y &= r sin\theta sin\varphi \\ h_z &= r cos\varphi \end{aligned} \quad (3.8)$$

Values of angles: θ, φ were selected as pseudorandom (Matlab's *rand*) ones from ranges of [-180°,180°] and [-90°,90°], respectively.

For the specified values of parameters presented in this section the simulation was carried out. Results of this simulation are reported in the next section.

Overview of results of the simulation

The main results of the carried-out simulation were the values of the estimated positioning errors estimated for the considered set of the measurement points. Fig. 4.1 presents how the determined values of the positioning errors varied during the simulation as a part of the assumed actual values of the errors (in %). The comparison of the selected values of the estimated positioning errors is presented in Tab. 4.1.

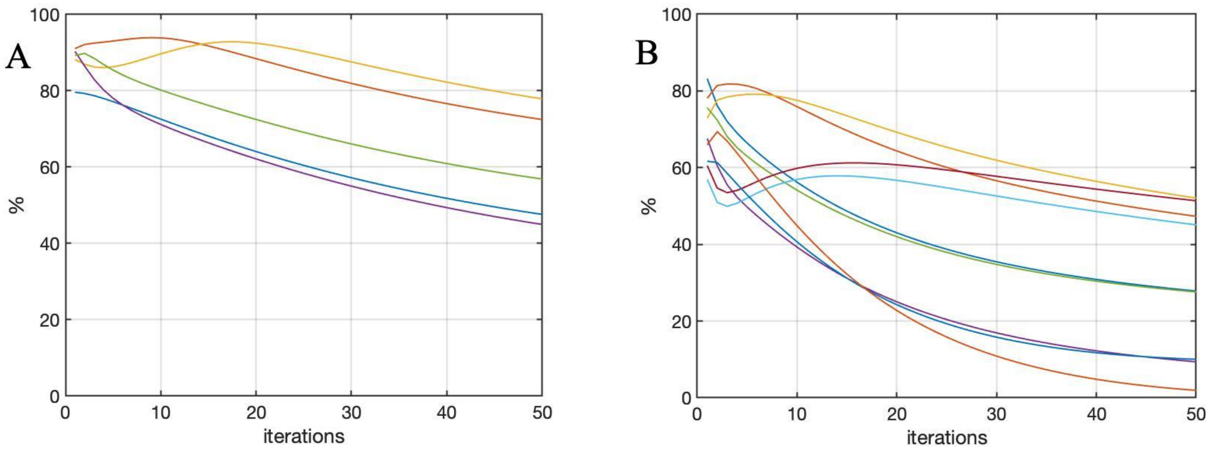

Fig. 4.1. Estimates of position errors relative to the actual values: A. 5 points, B. 9 points

Tab. 4.1. Comparison of values of positioning errors corresponding to the measurement points [mm]

	points	P1	P2	P3	P4	P5	P6	P7	P8	P9
Actual values		4.67	5.22	4.74	4.80	5.15	4.20	4.49	4.29	3.37
After 1000 iterations	9 points	4.53	4.81	4.45	5.15	4.74	4.09	4.03	3.65	3.43
After 50 iterations	5 points	2.45	1.44	1.05	2.64	2.22	-	-	-	-
	9 points	3.37	2.75	2.27	4.35	3.73	2.30	2.19	3.86	3.30

After 1 iteration	5 points	0.95	0.47	0.56	0.47	0.56	-	-	-	-
	9 points	0.78	1.14	1.28	1.56	1.25	1.81	1.77	1.64	2.12

The first conclusion coming out from the comparison of the estimation results was that the initial estimation of the positioning error was very coarse. The presented results proved that that application of identification of parameters of DH model (zero joint offsets), generally, improved the accuracy of estimation of the positioning errors. The effect was worse for 5 than for 9 measurement points. For the latter case, after 1000 iterations, estimation of positioning errors was considerably better than for 50 iterations and, except for P8 point, the difference between the actual and the estimated values did not exceed 10% of the actual value (about 0.5 mm).

The parameter identification was used in the conducted simulation only as a tool for improving estimates of the positioning errors. Fig. 4.2 presents the courses of the ratio of absolute values of the estimated zero joint offsets to their actual values (in %). Table 4.2 lists the obtained values of zero joint offsets.

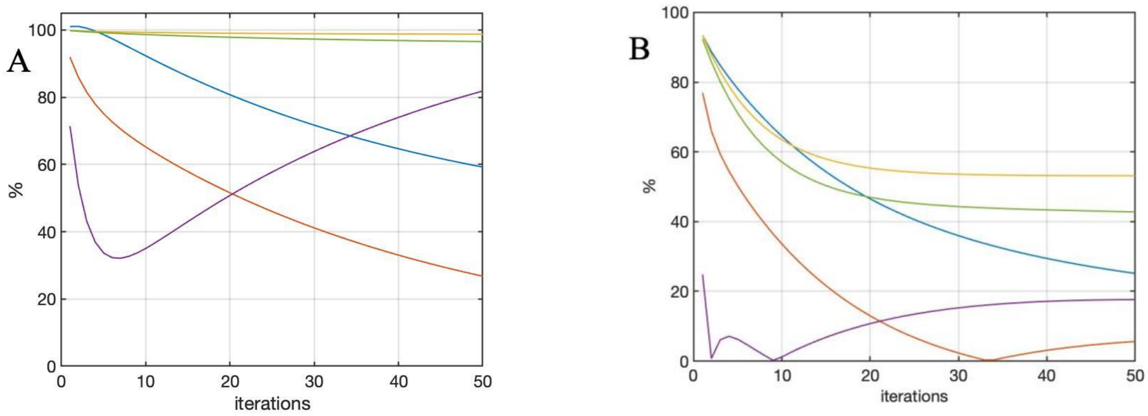

Fig. 4.2. Errors of estimation of values of zero joint offset relative to the actual values: A. 5 points, B. 9 points

Tab. 4.2. Comparison of values of zero joint offsets [rad]

Value	points	$\delta\theta_2$	$\delta\theta_3$	$\delta\theta_4$	$\delta\theta_5$	$\delta\theta_6$
Actual		0.00873	-0.00873	0.00873	0.00873	-0.00873
After 1000 iterations	9 points	0.00879	-0.00881	0.00406	0.00822	-0.00788
After 50 iterations	5 points	0.00356	-0.00639	0.00011	0.00158	-0.00030
	9 points	0.00653	-0.00921	0.00408	0.00719	-0.00489
After 1 iteration	5 points	-0.00009	-0.00070	0.00001	0.00249	-0.00002
	9 points	0.00057	-0.00201	0.00057	0.00656	-0.00067

For low number of iterations and for the case of 5 measurement points the differences between the estimated and the actual values of zero joint offsets were significant. For 9 measurement points and after 1000 iterations, except for the 4th joint variable, the estimation errors of zero joint offset were less than 10%.

The alignment of the estimated base frame to the instrument frame was characterized by calculation of the distance between the origin of the estimated base frame from the origin of the actual one. The variation of this distance in the course of the simulation is presented in Fig. 4.3.

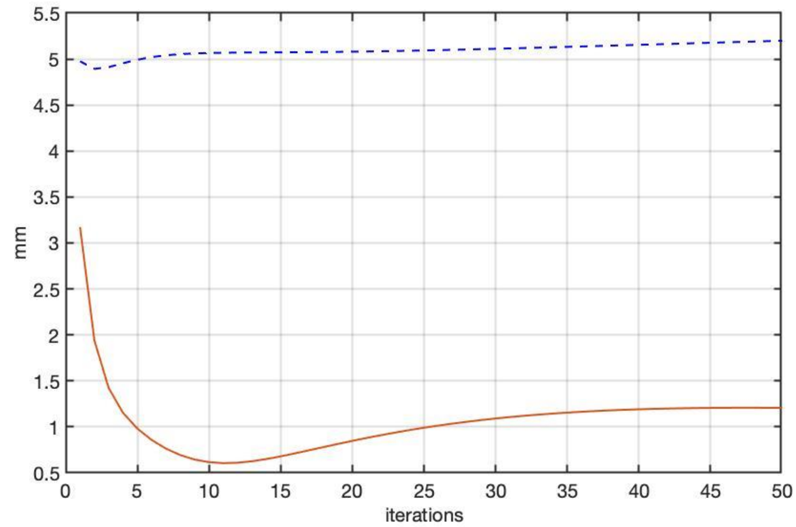

Fig. 4.3. Distance of the estimated location of the robot base frame from the actual one in the course of the simulation (lines: dashed – 5 points, solid – 9 points)

For the case of 9 measurement points, this value decreased significantly at the beginning of the simulation and later stabilized at about 0.5 mm (0.47 mm after 1000 iterations). This was presented in space in Fig. 4.4.

A B

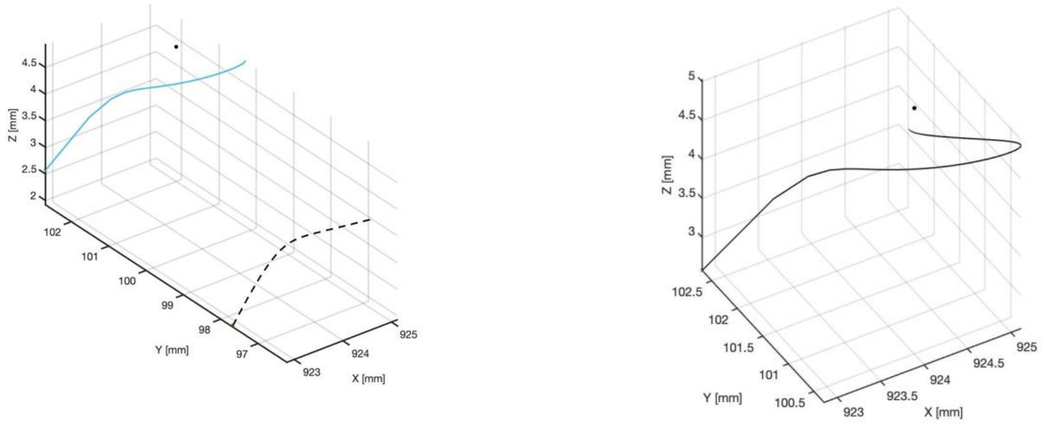

Fig. 4.4. Location of the actual and estimated origin of the robot base frame relative to the instrument frame (star – actual location, lines: dashed – 5 points, solid – 9 points) A. 50 iterations, B. 1000 iterations

The estimated distance of about 0.5 mm indicated that the introduced pseudorandom errors related to limited positioning repeatability and measurement accuracy, as being considerably smaller, proved to have negligible effect on position error determination Simulation results obtained in the result of simulations conducted for different setting of signs of errors (Tab. 3.4) of the geometrical parameters led to the qualitatively consistent results.

Final conclusions

As a result of the conducted simulation-based analyses of determination of positioning errors of robotic manipulators it was found that the initial estimates of the positioning errors may be considerably inaccurate, mainly as the result of the inaccurate alignment of the robot base frame and the reference measurement instrument coordinate frame. Next, it was found that using the standard 5 measurement points did not assure sufficiently accurate estimation of values of the positioning errors. The algorithm of determination of a set of 9 measurement points was suggested, which made obtaining more accurate estimates of positioning errors possible, and it did not increase significantly the effort made during determination of the measurement points due to the symmetrical positions of the cube vertices used in this procedure.

Application of parameter identification and updating of values of zero joint offsets in the iterative procedure of determination of the positioning errors considerably improved the accuracy of the estimation of the positioning errors.

The main limitation of the obtained results is related to that the conducted simulation was carried out for a single kinematic structure of a manipulator for the arbitrarily chosen deterministic error (the magnitude and distribution among parameters of DH models). Additionally, the applied method of estimation of zero joint offsets depended on the estimated alignment between the robot base frame and the instrument coordinate system, which could increase the estimation errors and spoil results of the elaborated iterative estimation procedure. Finally, the reported results of the simulation have not been confronted yet with the experimental results.

Acknowledgment

The research was funded by the statutory subsidy of the Faculty of Mechanical Engineering and Robotics, Department of Robotics and Mechatronics, AGH University of Krakow, Poland.

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