

Localization of Transmitting Devices in Multipath Environment Using MUSIC And Neural Networks*

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Abstract

This research examines the efficacy of the MUSIC algorithm in the context of electromagnetic wave propagation in urban environment. The Multiple Signal Classification (MUSIC) algorithm, known for its adaptability in direction-of-arrival estimation is analyzed for its efficacy in multipath propagation in urban areas, where reflections, diffractions and signal interactions with various obstacles may occur. The algorithm's practical applications for emitters localization in complex, multipath-dense situations are analyzed. The study contains insights into the algorithm's performance under various settings using simulated real-world scenarios. This research serves as proposal for the continued investigation of sophisticated localization methods in dynamic urban settings.

Keywords: multipath propagation, MUSIC algorithm, urban environments, localization.

Introduction

Localization

In contemporary wireless systems, localization technologies have emerged as essential elements across various application domains, such as autonomous navigation, robotics, augmented reality, logistics, emergency response and crisis management [N. Tayem, A. Duarte, A. Bochem, Y. Mao]. These systems facilitate high-precision locating capabilities and support context-aware services, operating reliably in both indoor and outdoor environments. Modern localization systems mostly depend on triangulation methods. These methods employ numerous spatially distributed sensing units that can estimate the Direction of Arrival (DOA) of incoming electromagnetic waves. By integrating DOA data from several receivers, along with precise knowledge of their spatial locations, one can accurately determine the coordinates of the signal emitter. These methods are especially efficient in open environments, such as rural areas, where the DOA reflects the signal source's direction, assuming that the emitter is stationary and the localization is estimated from a single snapshot rather than a temporal series of measurements conducted on the move. Thus, there is an increasing demand for the creation of more resilient and adaptable algorithms that can function in contested, congested, and dynamic settings.

Backward Raytracing (BR)

This work presents a new methodology utilizing back-reflection (BR) analysis predicted on approximated DOAs. This method's fundamental concept involves reconstructing potential signal propagation lines and estimation of the transmitter's position by pinpointing the intersections of these paths. A very comprehensive three-dimensional (3D) representation of the environment is essential for effectively modeling such a scenario. Furthermore, in urban environments, it is essential to integrate data regarding the physical characteristics and materials of existing structures and obstacles. This supplementary data facilitates the calculation of signal attenuation via path loss models, consequently permitting the system to limit the number of reflections analyzed depending on the material-specific loss attributes along each path. In many situations, the reconstructed pathways generally converge at a specific area, roughly aligning with the actual position of the transmitter. Conversely, metropolitan environments exhibit intricate signal behavior: propagation channels are no longer linear, and due to multiple reflections and diffractions, the reconstructed rays may converge at various points, frequently disconnected from the real emitter location. This spatial uncertainty considerably diminishes the efficacy of the BR method. The study uses artificial neural networks to tackle this issue. These networks are taught to evaluate the received signals and suggest the most probable area where the transmitter is situated. By doing so, the network restricts the solution space, enabling the system to eliminate additional crossing sites that fall beyond the anticipated region. Consequently, only intersections inside the designated area are taken into account, significantly enhancing localization precision. Notwithstanding the theoretical viability of this integrated Bayesian reasoning and neural network methodology, an additional significant problem appears: the identification and differentiation of distinct signal pathways. The majority of current DOA estimating algorithms are designed to identify solely the strongest incoming signal path—usually the line-of-sight (LOS) component or a predominant reflection. Accurate BR-based localization requires the identification of various unique pathways, including non-line-of-sight (NLOS) components, which are typically attenuated and hard to process using conventional methods. Addressing this constraint is crucial for the suggested technique to operate efficiently in densely populated, urban environment with many sources of radio signals.

MUSIC Algorithm

The aforementioned challenge prompts the evaluation of the MUSIC (Multiple Signal Classification) algorithm [M.M. Guncjal, J. Zhang, S. P. Drake, He Weigang, Z. -H. Liu, N. A. Lafta], which seems exceptionally appropriate for this situation. MUSIC is a high-resolution spectral estimating method that can detect many signal sources transmitting at the same frequency. The authors proved in previous work that from the algorithm's viewpoint, there is no intrinsic difference between signals emitted from various transmitters and instances of the identical signal arriving over diverse propagation paths. This positions it as a viable option for addressing multipath components necessary for urban localization. MUSIC functions by utilizing the mathematical framework of the covariance matrix obtained from signals captured by an array of spatially distributed sensors, such as antenna elements. The program first generates a spatial covariance matrix from the sampled sensor data. The matrix undergoes eigenvalue decomposition, yielding two orthogonal subspaces: the signal subspace, encompassing the principal eigenvectors linked to the real signals, and the noise subspace, comprising the residual eigenvectors associated with background noise. The core premise of MUSIC is based on the orthogonality between the noise subspace and the steering vectors associated with the actual Directions of Arrival (DOAs) of incoming wavefronts. The algorithm generates a spatial pseudo-spectrum by methodically scanning a specified angular domain and projecting candidate steering vectors onto the noise subspace. The peaks in this spectrum indicate the estimated DOAs of the incoming signals.

Objective of the research

The main aim of this research is to examine the effective use of BR of reflected electromagnetic signal paths to precisely ascertain the emitter's location in diverse metropolitan environments. Additionally, how will BR systems operate in intricate and varied metropolitan environments, where signal routes are non-line-of-sight, extensively distributed, and frequently subject to significant attenuation? This study delineates the methods utilized to tackle these inquiries, the simulated results acquired, and the conclusions derived from the research outcomes.

Methodology

Obtaining Data

For the purpose of simulation in the MATLAB environment, a virtual area measuring 600 meters by 550 meters was constructed. Within this space, 16 high-rise buildings were positioned at uniform intervals, emulating the regular grid-like urban layout characteristic of a Manhattan-type environment.

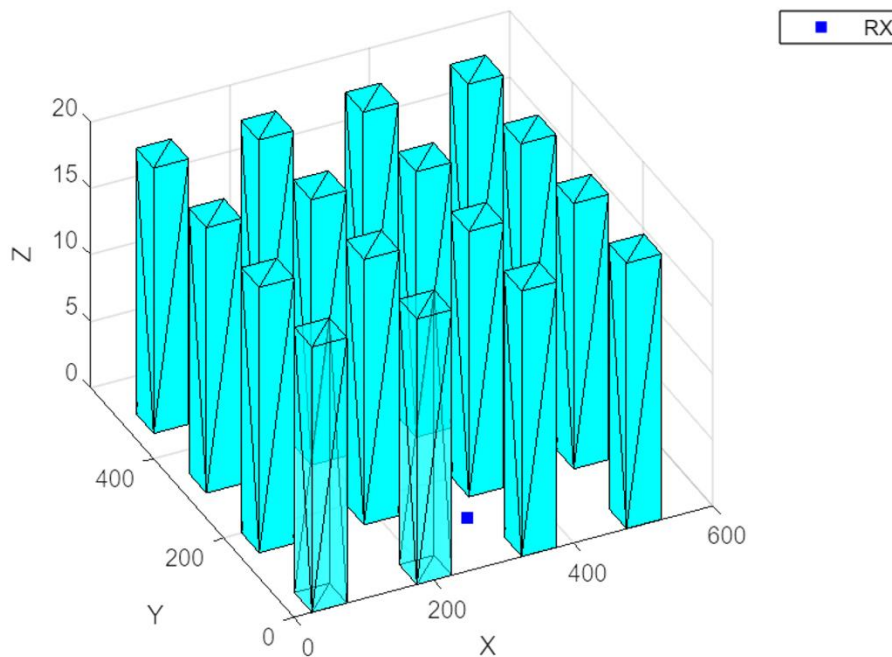


Fig. 1 Modelled terrain

The Rx position was fixed, whereas the Tx position was estimated for different locations. Subsequently, ray tracing was performed for all possible transmitter positions within the defined area, using a spatial resolution of one meter. This procedure enabled the extraction of detailed information regarding the directions of signal arrival, power losses along individual propagation paths, time delays associated with each path, as well as phase shifts induced by reflections. The acquisition of these propagation characteristics constitutes a crucial step, facilitating the transition to the next phase of the simulation-based research.

Processing Raytracing Data to MUSIC Algorithm

Based on the previously obtained simulation data, a signal matrix was generated for each transmitter location within the simulated environment. In this matrix, each column represented the time-domain signal corresponding to a specific transmitter position, while each row corresponded to a successive time sample. This structure allowed for a comprehensive representation of the changes of the signal as perceived at a fixed receiver position for various transmitter coordinates across the simulation grid [J. Gajewski].

To simulate the reception process in a linear antenna array configuration, the generated signal matrix was subsequently used as input to the CollectPlaneWave function. This function, in combination with the previously estimated angles of arrival (AoA) of the signal components, allowed for the spatial projection of the incoming wavefronts onto a Uniform Linear Array (ULA) composed of 11 equally spaced dipole antenna elements. As a result, a new matrix of received signals was obtained, in which each column contained the signal samples received by a specific antenna element, and each row represented a discrete time sample across the array aperture.

This configuration effectively captured the spatial diversity of the incoming signals and enabled further processing using high-resolution spectral estimation techniques. The MUSIC algorithm was applied to the resulting array output matrix. The MUSIC algorithm exploited the eigen structure of the signal's spatial covariance matrix to accurately estimate the directions of arrival of the multipath components.

Creating Neural Networks (NN)

Two artificial neural networks were developed to perform transmitter localization within the simulated environment. The first network partitioned the analysed area into 51 small sectors, enabling high-resolution localization, while the second network divided the same area into 6 bigger sectors to achieve a less precise, yet computationally more efficient classification.

The primary objective of both networks was to determine, based on the features extracted from the received signal at the fixed receiver, the most probable sector in which the transmitter was located. To achieve this, both neural models were trained using the previously generated dataset, which included representative signal patterns corresponding to various transmitter positions. The networks learned to associate these signal patterns with specific spatial regions, leveraging the temporal and spatial characteristics captured during the ray tracing and signal collection stages.

This supervised training process allowed the networks to generalize the complex relationship between the received signal properties and the transmitter's location, thus enabling sector-level localization of the signal source under varying propagation conditions.

The neural network dividing the area into 51 sectors achieved a classification accuracy of approximately 60%, whereas the network based on a 6-sector division reached an accuracy of 80%.

NNs structure

The Long Short-Term Memory (LSTM) architecture was selected for the classification task due to its capacity to effectively model time-dependent patterns in sequential data [D. A. Bibb, D. Pang, T. Abubakr, D. Qin, H. Wang]. From the receiver's perspective, it is possible to determine the time instances at which different multipath components arrive. This temporal differentiation was established by applying the MUSIC algorithm for an increasing number of time-domain samples. By observing the MUSIC spectrum as more samples were included, the appearance of additional signal paths over time was detected.

Given this time-resolved signal structure, an LSTM-based approach was particularly suitable. LSTM networks, a variant of Recurrent Neural Networks (RNNs), are specifically designed to retain information over extended time intervals and are capable of modelling complex temporal dependencies — a crucial feature in analyzing signal arrival patterns that unfold over time.

BR Realization

The data obtained from the initial ray tracing simulation and subsequently processed using the MUSIC algorithm were further utilized to perform reverse ray tracing in order to estimate the transmitter location. Prior to this, the dataset was subdivided based on the number of distinct signal paths observed at the receiver. This categorization was motivated by the assumption that the accuracy of localization improves with the number of propagation paths, as a richer multipath environment provides more spatial diversity and geometric constraints for estimating the source location.

In addition, measurements containing a Line-of-Sight (LOS) path were isolated and excluded from the main localization analysis. In a Manhattan-type urban environment, the presence of a LOS path typically implies that the transmitter is directly visible from the receiver, thus eliminating the need for non-cooperative localization algorithms. Moreover, LOS signals tend to dominate the received signal energy and can mask the presence of weaker multipath components, thereby degrading the resolution and reliability of algorithms such as MUSIC, which rely on the distinctiveness of spatial signatures [J. Gajewski].

Following this preprocessing, the BR procedure was carried out. For each estimated DoA, the corresponding propagation path was reconstructed by tracing the signal's trajectory backward through the environment, accounting for possible reflections from building surfaces. This process enabled the identification of intersection points between multiple reconstructed signal paths — that is, the estimated geometric points where multiple rays were most likely to converge, thus indicating the possible transmitter location.

To refine the estimation, the set of intersection points was post-processed through a filtering stage, in which outlier points that were significantly distant from the majority cluster were discarded. The remaining, spatially coherent intersection points were then averaged to produce a final estimate of the transmitter's location.

This approach leverages both physical modelling of the environment and data-driven signal processing, resulting in a robust and interpretable localization method that is particularly effective in complex urban scenarios characterized by rich multipath propagation.

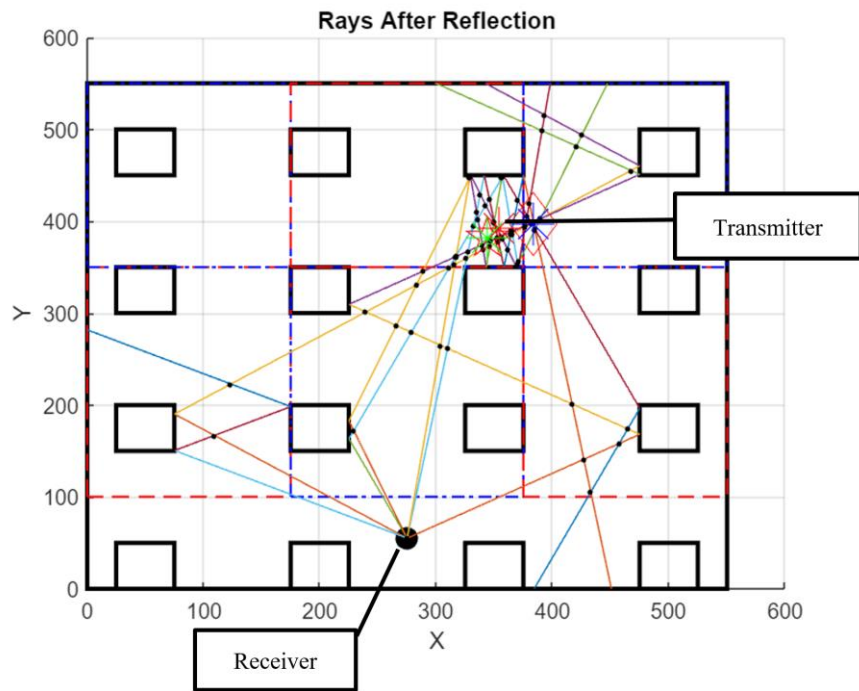


Fig. 2 BR illustration

Filtering data using NNs

To further improve the accuracy of the transmitter localization and to eliminate false intersection points arising from multiple crossing of signal paths, the previously trained neural networks were used as an additional filtering mechanism. Specifically, the predicted sector output by the neural network — whether from the 51-sector or the 6-sector model — was used to constrain the spatial domain in which valid intersection points could exist.

By discarding all intersection points that lay outside the sector predicted by the neural network, the method effectively reduced the influence of false intersection points.

As a result, only those intersection points consistent with both the ray-based propagation model and the neural network's sector-level classification were retained, and their averaged position was used as the refined estimate of the transmitter's location.

Results Comparison

The performance of the transmitter localization method was evaluated by comparing the results obtained under three different conditions:

- (1) using the neural network trained with 51 sectors,
- (2) using the neural network trained with 6 sectors,

(3) without any neural network assistance, relying solely on the raw intersection points of the back-projected rays derived from the MUSIC-based direction-of-arrival estimates.

A detailed analysis was conducted to examine how the localization accuracy varied as a function of the number of received signal paths. The underlying assumption was that an increased number of distinct multipath components would provide more geometric data and, consequently, enable more precise estimation of the transmitter's location.

The results demonstrate that incorporating the neural networks into the localization process improved the final position estimates, particularly in scenarios with a limited number of multipath components.

Results of the simulation

Simulation chart

The simulation was conducted in accordance with the schematic diagram shown below.

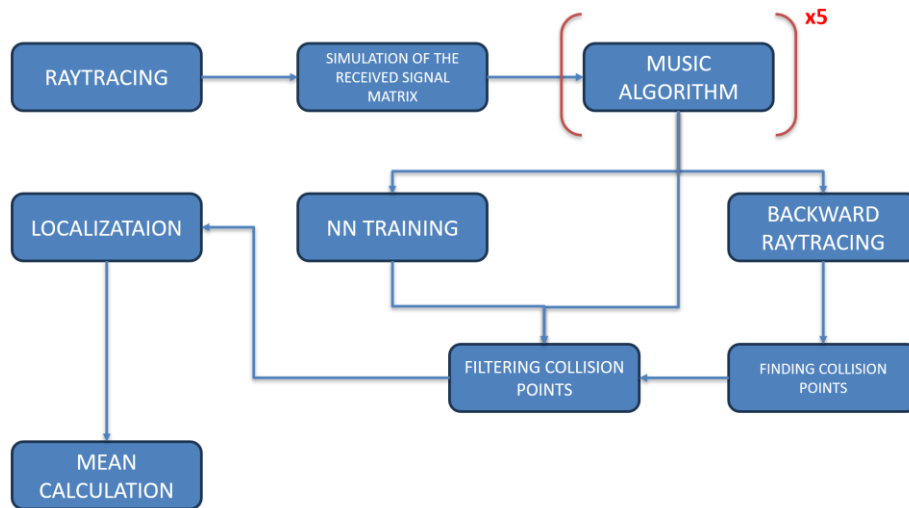


Fig. 3 Simulation chart

To obtain the input data, forward ray tracing was performed from the signal source to the receiver, as described in the earlier part of the article. These data were subsequently used to simulate the matrix of signals received by a ULA antenna configuration and were subjected to five independent runs of the MUSIC algorithm. This repeated application allowed the creation of training data that captured the variability in MUSIC's output, enabling the neural networks to learn and adapt to potential fluctuations in the algorithm's performance.

In the next step, the resulting dataset was used both to train the neural networks and to perform BR, aimed at reconstructing the signal paths and identifying their intersection points — interpreted as potential transmitter locations.

These intersection points were then filtered using the predictions of both neural networks. Each network determined the most probable sector in which the emission source was located, and only intersection points falling within the predicted sector were retained. In cases where no intersection points were found within the predicted sector, the centre of the sector was assumed as the estimated transmitter location.

Additionally, a third localization method was applied, utilizing a custom outlier rejection function based on the median distance between intersection points. This method removed spatially isolated points prior to final estimation.

Finally, the remaining intersection points, after filtering, were averaged to estimate the final location of the emission source. Furthermore, for each transmitter position, the final localization result was obtained by averaging the outcomes of the five MUSIC algorithm runs, providing a more stable and robust estimate.

Simulation Results

The maximum number of propagation paths through which the signal was able to reach the receiver was seven. However, such cases were rare (only 14 instances), which made a statistically meaningful analysis of localization errors impossible. Furthermore, the neural networks were not sufficiently trained for scenarios with such a high number of signal paths. As a result, the neural networks produced three significant outliers, although aside from these, their localization accuracy remained within 50 meters of the actual transmitter position.

The outlier rejection method based on the filtering of distant intersection points also failed to provide satisfactory results in this case, as the linear localization error fluctuated between 50 and 130 meters, which is considered unacceptable. This issue was likely caused by the excessive number of points where signal paths intersected along their propagation routes, leading to increased ambiguity in the localization process.

The localization error is illustrated in the following characteristic plot.

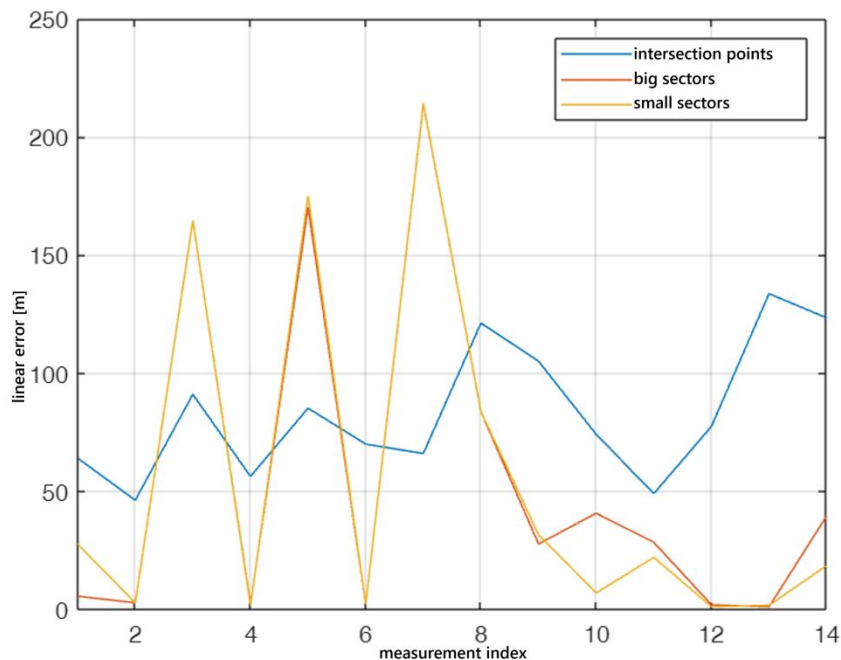


Fig. 4 Linear error for 7 rays received

For the subset of data corresponding to cases with six received signal paths, the advantage of the neural networks over the custom intersection-point filtering method becomes clearly evident. Although the dataset size (244 cases) was still insufficient to produce fully reliable results, it was large enough to allow for a meaningful statistical analysis.

The mean localization error for both networks—89.75 m for the network dividing the area into large sectors and 36.16 m for the network dividing the area into smaller sectors—was approximately equal to the median error in both cases, which indicates a balanced and symmetric error distribution. Moreover, the interquartile range for both cases was within 25–30 m, further confirming the stability and consistency of the neural network estimations.

By contrast, the custom method based on discarding distant intersection points failed to produce results of acceptable quality. In most cases, its outputs appeared to be randomly distributed rather than indicative of the true transmitter location.

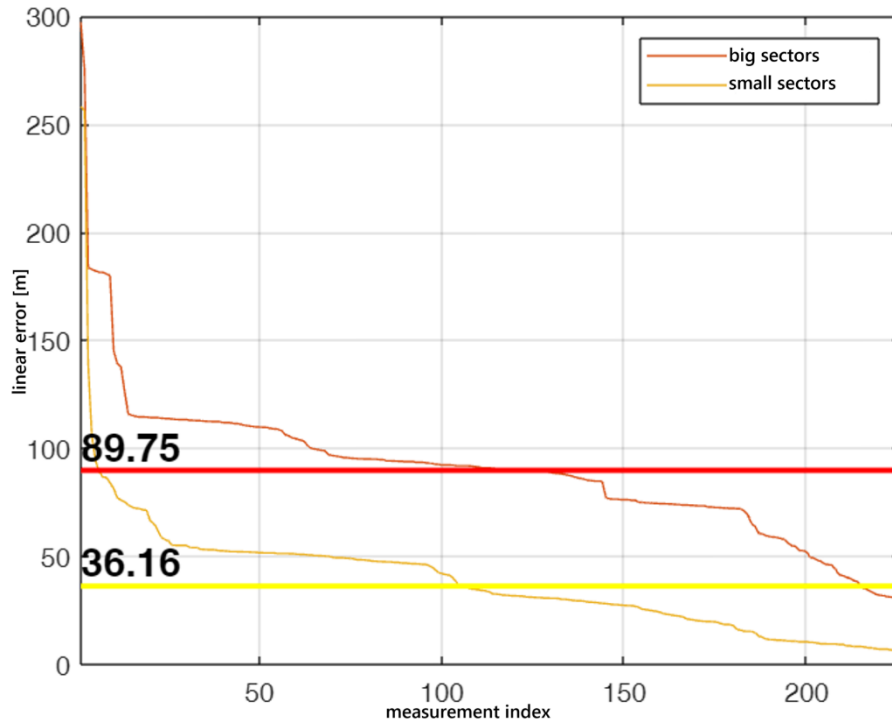


Fig. 5 Linear error for 6 rays received

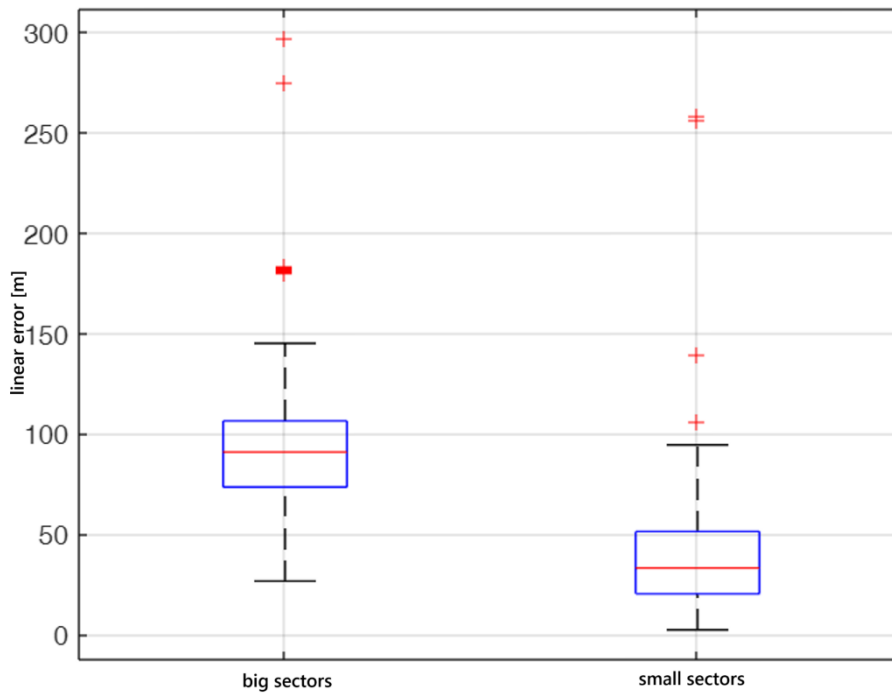


Fig. 6 Linear error for 6 rays (boxplot)

For the case of five paths, the amount of training data for the neural networks increased substantially, as the number of available cases exceeded 2,000. This larger dataset provided a solid foundation for improving the overall accuracy and reliability of the network predictions. Notably, the network that segmented the area into smaller sectors delivered particularly strong results. The mean error for this model was approximately 10.53 meters, which was significantly above the median value, indicating the presence of outliers or a skewed error distribution. Nevertheless, the interquartile range was exceptionally narrow, falling between 2 and 15 meters,

which highlights the model's impressive stability and the consistency of its predictions across most test scenarios.

However, it is worth emphasizing that a relatively high number of outliers was still observed. This indicates that, while the approach shows great promise and demonstrates high predictive power, it still requires additional refinement and tuning to achieve a fully robust and reliable performance, particularly in edge cases where the input data may be ambiguous or noisy.

Interestingly, the proprietary method based on discarding distant intersection points also began to yield correct and meaningful results. While this alternative approach demonstrates a certain degree of potential and improvement compared to earlier trials, it remains clearly inferior to the predictions generated by the neural network models, both in terms of average error and the stability of its outputs. The comparison between the two methods further reinforces the conclusion that neural networks, when provided with sufficient training data and well-chosen architectures, are capable of outperforming more traditional heuristic-based solutions in both accuracy and robustness.

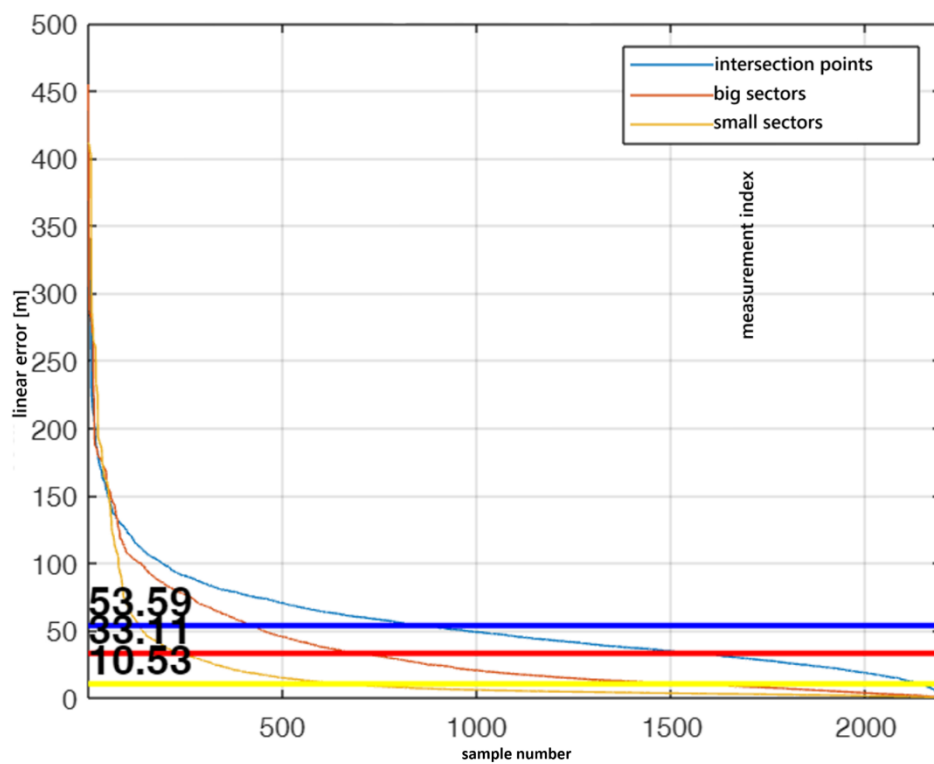


Fig. 7 Linear error for 5 rays received

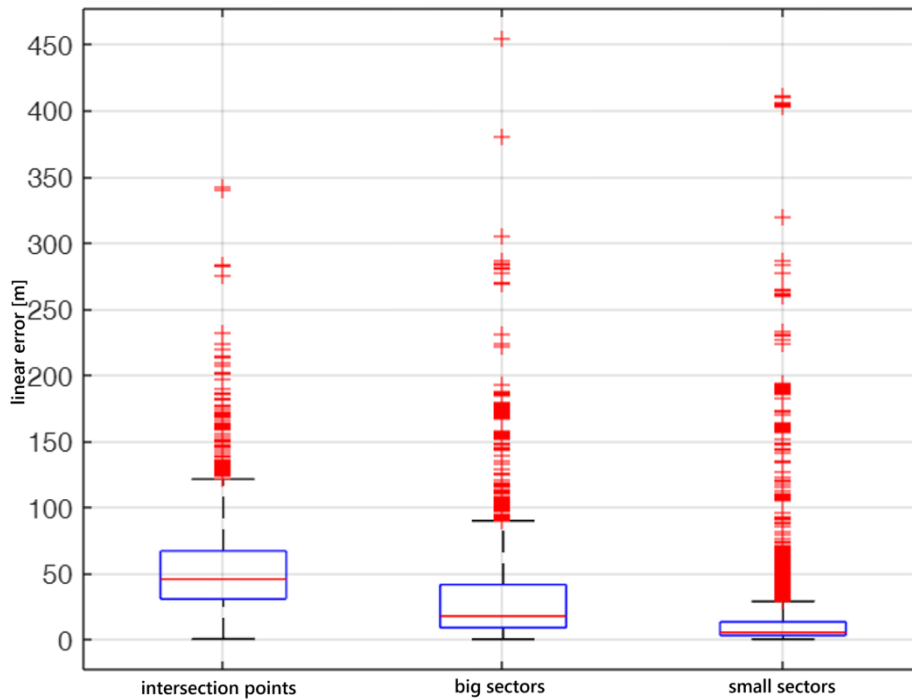


Fig. 8 Linear error for 5 rays (boxplot)

Simulation Analysis

For 6 paths, the results slightly increased compared to the 7 paths case, which was expected due to more data samples. Nonetheless, the neural networks maintained a high level of accuracy, with the sector-based architecture again providing the most stable results. Although a slight increase in the interquartile range was observed, the error distribution remained narrow enough to confirm the robustness of the approach.

For 5 paths, a substantial increase in the number of training samples (over 2,000 cases) allowed the neural networks to compensate for the reduced input data. The sector-based network achieved a mean error of 10.53 m, with an interquartile range of 2–15 m, indicating stable predictions. However, the presence of a noticeable number of outliers suggests that further refinement of the model is necessary. The proprietary method based on discarding distant intersection points also began to yield correct results but still performed worse than the neural networks.

In summary, the experiments confirm that neural network-based approaches outperform heuristic methods, especially when sufficient training data is available. The sector-based architecture stands out as the most reliable solution, maintaining stable performance even with a reduced number of radii, though additional optimization could further minimize outliers.

Discussion

A more significant parameter than the number of signal paths reaching the receiver turned out to be the size of the dataset available for a given number of radii, which can be used to train the neural networks. Only a sufficiently large dataset can ensure reliable predictions by the networks, leading to a considerable improvement in localization accuracy in urban environment.

At the same time, the decreasing number of signal paths allowed for more accurate position determination using the function designed to discard distant intersection points. This is likely due to the fact that a larger number of signal paths results in a high number of intersection points, but their relative density around the true source location decreases, which makes it harder to pinpoint the correct area.

Conclusions

The use of additional receiving devices would enable to obtain a larger number of signal paths, likely improving the density of intersection points around the transmitter location. This would allow the algorithm to operate more effectively.

It is also necessary to conduct experiments on tuning the hyperparameters of the neural networks, which could help improve their performance and ensure more stable and accurate predictions.

An interesting alternative could be shifting the analysis space into three dimensions, which could significantly increase the number of signal paths and improve localization accuracy.

Further works will be devoted to the experiments in a real-world environment, which will allow verification of the correctness of the simulation results.

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