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Directional Alternative Hypotheses in Data Homogeneity Analysis*

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Abstract

This paper deals with the use of permutation tests in the analysis of data homogeneity. The proposed solution is based on a nonparametric approach that involves the use of a combined permutation test. The strategy corresponds to Pesarin's Nonparametric Combination (NPC) method. This non-classical approach to statistical inference allows for the formulation of complex alternative hypotheses.
The paper presents a proposal for testing directional hypotheses based on data presented in contingency tables. The proposed method does not have the limitations of the chi-square test regarding the expected frequencies in the cells of the contingency table. Moreover, in the case of a permutation homogeneity test with complex directional alternative hypotheses, the interpretation of results is more detailed than in the chi-square homogeneity test.
A two-step algorithm of the complex permutation procedure is used to assess the overall achieved significance level (*ASL*). The applied nonparametric statistical inference procedure uses a combining function. A simulation study was conducted to determine the size and power of the test. A Monte Carlo simulation was used to compare the empirical power of tests with different forms of combining functions. The advantage of the proposed method is its applicability even for small sample sizes. The idea of the proposed method is illustrated using data from the European Social Survey (ESS) project.

Keywords: permutation tests, homogeneity analysis, contingency table, directional hypothesis, Monte Carlo simulation.

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Introduction

When verifying statistical hypotheses based on data presented in the form of contingency tables, the independence test and the homogeneity test are most used. The chi-square independence test can be employed if a single sample is categorized on two variables. This test assumes that the sample is randomly selected from the population. The chi-square test for homogeneity is applicable when r independent samples are classified based on a single variable with c categories ($r \geq 2$, $c \geq 2$). It is assumed that the sums of the r rows correspond to the total number of observations in each sample.

Statistical inference based on data in contingency tables requires that the expected counts in each cells of the contingency table must be at least 5 (Agresti, 1996; Sheskin, 2004). If this condition is not met, methods incorporating Yates' correction and Dandekar's correction can be used for 2 × 2 tables (Yates, 1934; Rao, 1973). It is also possible to use Fisher's exact test (Fisher, 1935). In the case of $r \times c$ contingency table ($r > 2$, $c > 2$), permutation tests can be used (Kończak, Chmielińska, 2013). The analysis of two-way tables can be extended to the multi-way tables (Polko, Kończak, 2014).

In data homogeneity analysis in contingency tables with two rows and two columns, it is possible to employ a directional (one-tailed) alternative hypothesis (Good, 2006). When evaluating contingency tables with r rows ($r > 2$) and c columns ($c > 2$) then directional multi-tailed alternative hypothesis can be employed (Kończak, 2012).

The aim of this article is to present a proposal for a statistical test in the analysis of data homogeneity. The proposed solution involves using a complex permutation test for data in contingency tables. The strategy proceeds in two stages and corresponds to Pesarin's nonparametric combination method (NPC). This non-classical method of statistical inference allows for the formulation of complex alternative hypotheses, including directional ones. The procedure for evaluating the overall *ASL* (*achieved significance level*) value relies on the use of combining functions. The method is based on a test procedure using permutations of the data set and does not impose restrictions on the form of the distribution of the variable in the population, so it is also applicable in the case of small sample sizes. A simulation study was performed to evaluate the empirical size and power of the test.
The proposed statistical test can be widely used in the statistical analyses of multidimensional socio-economics phenomena. Such analyses may include examining gender equality across different industries, comparing employment structures across various sectors, analyzing brand or product type preferences among different income groups, or comparing voting preferences across social groups. In the paper the idea of the proposed method is illustrated with an example. The data is taken from the European Social Survey (ESS) project.

The homogeneity tests for two-way contingency tables

Let us consider independent samples drawn from r populations, and let Y denote the population from which each independent sample was taken. Let X denote a categorical variable having c levels. Table 1 presents the model for the data from r populations. The entries n_{ij} ($i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, c$) are the counts for every two-way combination of i^{th} row and j^{th} column. The total number of observations in table 1 is equal to

$$n = \sum_{i=1}^{r}\sum_{j=1}^{c} n_{ij} = \sum_{i=1}^{r} n_{i\cdot} = \sum_{j=1}^{c} n_{\cdot j} \ .$$

Table 1: Contingency table (*r* populations)

Population *Y*	Variable *X*				Row sums
	x_1	x_2	…	x_c	
1	n_{11}	n_{12}	…	n_{1c}	$n_{1\cdot}$
2	n_{21}	n_{22}	…	n_{2c}	$n_{2\cdot}$
…	…	…	…	…	…
r	n_{r1}	n_{r2}	…	n_{rc}	$n_{r\cdot}$
Column sums	$n_{\cdot 1}$	$n_{\cdot 2}$	…	$n_{\cdot c}$	n

Source: own work.

The distribution of the categorical variable for each population is presented in table 2.

Table 2: The probabilities in contingency table

Population *Y*	**Variable *X***				Row sums
	x_1	x_2	…	x_c	
1	p_{11}	p_{12}	…	p_{1c}	1
2	p_{21}	p_{22}	…	p_{2c}	1
…	…	…	…	…	…
r	p_{r1}	p_{r2}	…	p_{rc}	1
Column sums	$p_{\cdot 1}$	$p_{\cdot 2}$	…	$p_{\cdot c}$	1

Source: own work.

The value p_{ij} represents the probability that the observation from i^{th} population is equal to x_j. This probability can be written $p_{ij} = P(X = x_j | Y = i)$. The probabilities p_{ij} are usually unknown and can be estimated using the following formula

$$\hat{p}_{ij} = \frac{n_{ij}}{n_{i\cdot}} ,$$

where

n_{ij} for $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, c$ are the observed counts,
$n_{i\cdot}$ for $i = 1, 2, \ldots, r$ are the number of observations form i^{th} population.

The null hypothesis and the alternative hypothesis in homogeneity testing is usually formulated:

H_0: In the underlying populations, the proportions in each column of the $r \times c$ contingency table are equal.
H_1: In the underlying populations, the proportions in at least one column of the contingency table are not equal.
Formally these hypotheses can be written as follows:

$$\begin{aligned} H_0 :\ & p_{11} = p_{21} = \cdots = p_{r1} \\ & p_{12} = p_{22} = \cdots = p_{r2} \\ & \cdots \\ & p_{1c} = p_{2c} = \cdots = p_{rc} \end{aligned} \tag{1}$$

and the alternative

$$\boldsymbol{H_1 : p_{ij} \neq p_{kj}} \tag{2}$$

for some $i, k = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, c$.
It could be used chi-square statistic to test the hypothesis H_0 against the above alternative

$$\boldsymbol{\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left(n_{ij} - \hat{n}_{ij}\right)^2}{\hat{n}_{ij}}} \tag{3}$$

where $\hat{n}_{ij}$ for $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, c$ are the expected counts ($\hat{n}_{ij} = \frac{n_{i\cdot} \cdot n_{\cdot j}}{n}$).

Statistic (3) has asymptotic chi-square distribution with $(r-1)(c-1)$ degrees of freedom. The chi-square statistic (3) is applicable for testing the hypothesis (1) against (2) only when the expected count in each cell of the contingency table is 5 or greater.

In addition, researchers are usually more interested in the directional form of the alternative hypothesis. To test the hypothesis (1) against the directional alternative hypothesis the permutation test can be used. Let us consider the hypothesis (1) with the alternative (Polko, Kończak, 2016; Polko-Zając, 2021)

$$\boldsymbol{H_1 :\ p_{ij} > p_{kj}} \tag{4}$$

for some $i, k = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, c$.

In permutation testing the principal issue is deciding on an appropriate test statistic that discriminates between the null hypothesis and the alternative. The test statistic can be written

$$\boldsymbol{T} = \widehat{\boldsymbol{p}}_{\boldsymbol{ij}} - \widehat{\boldsymbol{p}}_{\boldsymbol{kj}}. \tag{5}$$

The procedure for performing the permutation test is outlined below:

1. Assume the significance level α.
2. Compute the test statistic (T_0) using the sample data.
3. Generate N permutations of the dataset and calculate the test statistic (T_i) for i^{th} permutation.
4. Estimate the ASL value using the empirical distribution of the test statistic

$$\widehat{ASL} = \frac{card\{i{:}T_i \geq T_0\}}{N}.$$

If ASL is less than α, the H_0 hypothesis is rejected, otherwise there is no reason to reject the H_0 hypothesis.

The alternative hypothesis given in (4) is one-directional. With the use of permutation test, it is possible to consider a multi-directional form of this hypothesis (see Kończak, 2012). The issue of testing complex, directional hypotheses can also be considered using a strategy corresponding to the non-parametric combination method proposed by Pesarin (2001). The non-classical procedure for assessing the ASL value is conducted in two stages. The null hypothesis (1) that the structures in r populations are homogeneous is tested against the complex alternative hypothesis

$$\boldsymbol{H_1} \ : \boldsymbol{H_1^{(1)}} \vee \ \boldsymbol{H_1^{(2)}}, \tag{6}$$

which is decompose into

$$H_1^{(1)}{:}\ p_{ij} > p_{kj}\ ,\ \ H_1^{(2)}{:}\ p_{ut} > p_{vt}. \tag{7}$$

for some $i, k, u, v = 1, 2, \ldots, r$ and $j, t = 1, 2, \ldots, c$.
The test statistics can be written as follows

$$T^{(1)} = \hat{p}_{ij} - \hat{p}_{kj} \quad \text{and} \quad T^{(2)} = \hat{p}_{ut} - \hat{p}_{vt}. \tag{8}$$

In the initial phase of individually testing each of the partial hypotheses under consideration, the ASL values are calculated as follows:

1. Assume the significance level α.
2. Calculate the statistic ($T_0^{(1)}$and $T_0^{(2)}$) values using the sample data.
3. Perform the permutation of data N-times, then calculate the statistics test values ($T_i^{(1)}$and $T_i^{(2)}$) in i^{th} permutation.
4. Estimate the ASL values using the empirical distribution of the test statistics:
5.

$$\widehat{ASL}_{T^{(1)}}\left(T_0^{(1)}\right) = \frac{0.5 + card\left\{i{:}T_i^{(1)} \geq T_0^{(1)}\right\}}{N+1} \quad \text{and} \quad \widehat{ASL}_{T^{(2)}}\left(T_0^{(2)}\right) = \frac{0.5 + card\left\{i{:}T_i^{(2)} \geq T_0^{(2)}\right\}}{N+1}.$$

In the standard permutation ASL estimation, 0.5 is added to the numerator and 1 to the denominator of the fraction. This adjustment ensures that the estimated ASL values are contained within the open interval (0,1), preventing potential computational problems that may occur in the second step of the nonparametric procedure. However, since a large N is used, this correction is irrelevant (Marozzi, 2008).

The second stage of the nonparametric statistical inference procedure involves the determination of the overall ASL value using combining functions (Pesarin, 2001)

$${}_{\varphi 12}T = \varphi\left(ASL_{T^{(1)}}, ASL_{T^{(2)}}\right). \tag{9}$$

There are various forms of combining functions used to determine the overall ASL value, but the authors most commonly refer to the following functions (Polko-Zając, 2019):

- the Fisher omnibus combining function (Fisher, 1932)

$$C^{(F)} = -2\left[\log\left(\widehat{ASL}_{T^{(1)}}\right) + \log\left(\widehat{ASL}_{T^{(2)}}\right)\right], \tag{10}$$

- the Liptak combining function (Liptak, 1958)

$$C^{(L)} = \Phi^{-1}\left(1 - \widehat{ASL}_{T^{(1)}}\right) + \Phi^{-1}\left(1 - \widehat{ASL}_{T^{(2)}}\right), \tag{11}$$

where Φ denotes the standard normal distribution function,

- the Tippett combining function (Tippett, 1931)

$$C^{(T)} = max\{1 - \widehat{ASL}_{T^{(1)}}, 1 - \widehat{ASL}_{T^{(2)}}\}. \tag{12}$$

The value of the observed statistic for the sample data can be calculated as

$$_{\varphi 12}T_0 = \varphi\left(\widehat{ASL}_{T^{(1)}}(T_0^{(1)}), \widehat{ASL}_{T^{(2)}}(T_0^{(2)})\right), \tag{13}$$

while the distribution of this statistic is determined using the same permutations as in the first stage, for example, for the i–th permutation

$$_{\varphi 12}T_i = \varphi\left(\widehat{ASL}_{T^{(1)}}(T_i^{(1)}), \widehat{ASL}_{T^{(2)}}(T_i^{(2)})\right). \tag{14}$$

The overall ASL value for the test under consideration is estimated using the formula

$$\widehat{ASL}_{_{\varphi 12}T} = \frac{card\{i: {}_{\varphi 12}T_i \geq {}_{\varphi 12}T_0\}}{N}. \tag{15}$$

If $ASL < \alpha$, the hypothesis H_0 is rejected, otherwise there is no basis for rejecting the H_0 hypothesis.

Monte Carlo simulation

Using the nonparametric procedure based on combining functions, the size and power of the test were determined through simulation. The Monte Carlo simulation was conducted in R program. In the analyses, three populations with multinomial distributions and probability vectors $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{p_3}$ were compared, with samples taken of sizes n_1, n_2 and n_3. Multinomial distributions with three variants of the characteristic were considered.

As a result of the simulations, the contingency table with dimensions of 3 rows (populations) and 3 columns (variants of the variable X) was obtained. The following probability vectors were considered in the conducted simulations:

$\mathbf{p_a} = (x, \frac{1-x}{2}, \frac{1-x}{2})$, $\mathbf{p_b} = (\frac{x}{2}, 1-x, \frac{x}{2})$, $\mathbf{p_c} = (\frac{x}{2}, \frac{x}{2}, 1-x)$,
where $x \in \langle 0.1, 0.9 \rangle$ with a step of 0.05.

In the simulations, samples of sizes

a) $(n_1, n_2, n_3) = (20,20,20)$,
b) $(n_1, n_2, n_3) = (40,40,40)$,
c) $(n_1, n_2, n_3) = (20,30,40)$

were generated. In the first part of the study, homogeneous data were generated with $\mathbf{p_1} = \mathbf{p_2} = \mathbf{p_3}$ to evaluate the test size. In the second part of the simulations, samples were generated from heterogeneous populations with $\mathbf{p_1} \neq \mathbf{p_2} \neq \mathbf{p_3}$ to evaluate the power of the test under consideration.

Table 3: Estimation of the size of permutation test (for x=0.3)

Probability distribution	Combining function	(n_1, n_2, n_3)		
		(20,20,20)	(40,40,40)	(20,30,40)
$\mathbf{p_a}$	Fisher	0.049	0.045	0.054
	Liptak	0.055	0.049	0.056
	Tippett	0.040	0.044	0.047
$\mathbf{p_b}$	Fisher	0.050	0.055	0.056
	Liptak	0.050	0.055	0.058
	Tippett	0.040	0.044	0.053
$\mathbf{p_c}$	Fisher	0.055	0.041	0.057
	Liptak	0.051	0.040	0.058
	Tippett	0.042	0.048	0.046

Source: own calculations using computer simulations in the R program.

Table 4: Estimation of the power of permutation test

x	(n_1, n_2, n_3)								
	(20,20,20)			(40,40,40)			(20,30,40)		
	Combining function								
	Fisher	Liptak	Tippett	Fisher	Liptak	Tippett	Fisher	Liptak	Tippett
0.10	0.856	0.729	0.865	0.993	0.925	0.994	0.914	0.790	0.936
0.15	0.829	0.747	0.838	0.984	0.927	0.988	0.873	0.795	0.875
0.20	0.778	0.710	0.758	0.974	0.946	0.972	0.855	0.801	0.850
0.25	0.754	0.722	0.714	0.954	0.934	0.941	0.823	0.793	0.806
0.30	0.728	0.703	0.679	0.924	0.903	0.902	0.793	0.785	0.758
0.35	0.722	0.723	0.642	0.920	0.919	0.885	0.787	0.780	0.733
0.40	0.700	0.702	0.618	0.918	0.924	0.859	0.781	0.773	0.719
0.45	0.684	0.687	0.590	0.911	0.915	0.853	0.747	0.756	0.655
0.50	0.658	0.668	0.558	0.902	0.909	0.835	0.758	0.766	0.675
0.55	0.686	0.695	0.593	0.894	0.901	0.839	0.751	0.765	0.670
0.60	0.658	0.663	0.587	0.900	0.903	0.866	0.754	0.759	0.689
0.65	0.676	0.669	0.600	0.922	0.917	0.885	0.775	0.761	0.731
0.70	0.698	0.684	0.644	0.940	0.924	0.914	0.755	0.739	0.730
0.75	0.721	0.689	0.699	0.931	0.912	0.934	0.804	0.760	0.799
0.80	0.755	0.695	0.746	0.959	0.928	0.971	0.850	0.789	0.878
0.85	0.808	0.726	0.832	0.978	0.934	0.981	0.893	0.802	0.915
0.90	0.842	0.725	0.888	0.985	0.942	0.994	0.939	0.818	0.959

Source: own calculations using computer simulations in the R program.

The procedure for evaluating the properties of the proposed test involved 1000 Monte Carlo simulations and 1000 data permutations, with a specified significance level of α=0.05. The results of the simulations conducted to determine the test size and test power are presented in table 3 and table 4, respectively.

In the analysis of both equinumerous and non-equinumerous samples, the sizes of the presented tests were close to the assumed significance level. The estimated probabilities of rejecting the null hypothesis H_0 when it was true differed only slightly from α=0.05. The test, which considered three types of combining functions, achieved comparable assessments of the probabilities of rejecting the H_0 hypothesis when it was false. However, in most of the analyzed cases, the most powerful test was the proposed permutation test, which used a two-stage *ASL* determination method with the Fisher combining function.

Empirical example

An example illustrates the application of the proposed method. Table 5 presents collected data from European Social Survey in a contingency table (https://www.europeansocialsurvey.org/). The data contains a feeling about household income nowadays. The study was conducted from 2020 to 2022. Included in the table data related to five selected EU countries. The degree of satisfaction of current household income is defined on four levels: living comfortably on present income, coping on present income, difficult on present income, very difficult on present income.

Table 5: Satisfaction of household's income in selected EU countries in 2020-2022 (empirical conditional probabilities)

Country	**Feeling about household's income nowadays**			
	Living comfortably on present income	Coping on present income	Difficult on present income	Very difficult on present income
Italy	792 (31.1%)	1293 (50.7%)	389 (15.3%)	74 (2.9%)
France	757 (38.4%)	944 (47.9%)	238 (12.1%)	32 (1.6%)
Croatia	524 (33.4%)	814 (51.9%)	188 (12.0%)	42 (2.7%)

Greece	200 (7.2%)	1085 (39.2%)	1108 (40.1%)	374 (13.5%)
Portugal	292 (16.0%)	953 (52.4%)	407 (22.4%)	168 (9.2%)

Source: own work based on data from the European Social Survey.

The null hypothesis (1) in homogeneity testing stated as follows: “in all surveyed countries the samples represent all of the proportions of persons with degree of satisfaction of household’s income are the same” was being verified, towards three complex, directional alternative hypotheses:

a) $H_{1A}^{(1)}:\ p_{11} > p_{51}\ \vee\ H_{1A}^{(2)}:\ p_{12} > p_{52}$, that “proportions of persons that live comfortable on present income is greater in Italy than in Portugal or proportions of persons that cope on present income is greater in Italy than in Portugal”;
b) $H_{1B}^{(1)}:\ p_{33} > p_{23}\ \vee\ H_{1B}^{(2)}:\ p_{34} > p_{24}$, that “proportions of persons that find it difficult on present income is greater in Croatia than in France or proportions of persons that find it very difficult on present income is greater in Croatia than in France”;
c) $H_{1C}^{(1)}:\ p_{51} > p_{41}\ \vee\ H_{1C}^{(2)}:\ p_{22} > p_{42}$, that “proportions of persons that live comfortable on present income is greater in Portugal than in Greece or proportions of persons that cope on present income is greater in France than in Greece”.

Table 6: Calculation results

Example of directional hypotheses			A	B	C
First comparison	Compared countries (populations)		Italy - Portugal	Croatia – France	Portugal - Greece
	Selected category (column variable)		Living comfortably on present income	Difficult on present income	Living comfortably on present income
	Test statistics ($T_0^{(1)}$)		0.1504	-0.0009	0.0882
Second comparison	Compared countries (populations)		Italy - Portugal	Croatia – France	France - Greece
	Selected category (column variable)		Coping on present income	Very difficult on present income	Coping on present income
	Test statistics ($T_0^{(2)}$)		-0.0162	0.0106	0.0868
Overall *ASL* value		Fisher	0.0000	0.2053	0.0000
		Liptak	0.0017	0.1777	0.0000
		Tippett	0.0001	0.2022	0.0000
Decision			Reject H_0	No basis for rejection H_0	Reject H_0

Source: own calculations.

All calculations were carried out in the R statistical computing environment. A significance level $\alpha = 0.05$ in all performed tests was assumed. N=10000 permutations of data were used. The results of the calculations are presented in table 6. Permutation tests for three selected complex, directional alternative hypothesis were performed. The formula (8) was used as test statistics. In subsequent cases of considered alternative hypotheses, the test statistics can be written as

$$T_A^{(1)} = \hat{p}_{11} - \hat{p}_{51} \quad \text{and} \quad T_A^{(2)} = \hat{p}_{12} - \hat{p}_{52}, \tag{16}$$
$$T_B^{(1)} = \hat{p}_{33} - \hat{p}_{23} \quad \text{and} \quad T_B^{(2)} = \hat{p}_{34} - \hat{p}_{24}, \tag{17}$$
$$T_C^{(1)} = \hat{p}_{51} - \hat{p}_{41} \quad \text{and} \quad T_C^{(2)} = \hat{p}_{22} - \hat{p}_{42}. \tag{18}$$

The overall *ASL* values for hypothesis on data homogeneity with the alternative hypothesis formulated according to the first comparison of countries Italy and Portugal and according to the third comparison of countries: Portugal – Greece and France – Greece are lower than assumed significance level α. In these cases, the hypothesis on homogeneity H_0 is rejected in favour of hypothesis H_1. In the second case (comparison of countries: Croatia – France) the overall *ASL* values calculated with empirical distributions of statistics with the use of three combining function are higher than assumed significance level α therefore hypothesis H_0 cannot be rejected. Calculations results indicate that:

a) in Italy "living comfortably on present income" category occurs more often than in Portugal or in Italy "coping on present income" category occurs more often than in Portugal;
b) there is no basis for rejection null hypothesis that "in the countries the samples represent all of the proportions of persons with the level of satisfaction of present household income are the same";
c) in Portugal "living comfortably on present income" category occurs more often than in Greece or in France "coping on present income" category occurs more often than in Greece.

Conclusions

The permutation procedure for testing homogeneity based on data presented in contingency tables is proposed in the paper. The nonparametric combination method (NPC) described by Pesarin is used, allowing for the formulation of complex directional hypotheses. The proposed method does not have the limitations of the chi-square test regarding the expected frequencies in the cells of the contingency table. In addition, in the case of the permutation homogeneity test with complex directional alternative hypotheses, the interpretation of results is more detailed than in the chi-square homogeneity test. Rejecting the null hypothesis in the chi-square test suggests that the data are not homogeneous, but it does not indicate which specific categories differ.

Monte Carlo simulation enables comparison of the empirical power of tests using different form of combining functions. The testing procedure effectively controlled the Type I error at the designated significance level. The highest test power was attained by employing a nonparametric approach that uses Fisher's combining functions to evaluate the overall *ASL* value. One of the key advantage of the proposed method is its applicability even with small sample sizes. The idea of the proposed method is illustrated in an example. The data is taken from the European Social Survey (ESS) project.

The permutation test method provides a flexible tool for assessing the homogeneity of economic and social data, particularly in situations where the assumptions of other methods regarding data distribution or sample size are not met. A limitation of the proposed permutation test is the lack of clear criteria for selecting an appropriate combining function, which may lead to varying test results. However, based on the conducted study, the use of Fisher's combining function in the permutation testing procedure proved to be the most effective approach.

Possible future research directions, including evaluating the effectiveness of the proposed permutation test in cases involving more complex alternative hypotheses or multidimensional data analysis, as well as conducting detailed studies on selecting the optimal combining function.

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