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Unifying Aleatoric and Epistemic Uncertainty in Investment Evaluation*

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Abstract

An important problem in risk analysis is the distinction between aleatoric (probabilistic) uncertainty and epistemic uncertainty (resulting from a lack of precision or lack of information). The article describes the problem of making an investment decision based on the analysis of the profitability and risk of a tangible investment, in a situation where some input parameters of the decision model are presented as probability distributions and some as possibility distributions, in other words, in a situation of hybrid data. The risk analysis for such a defined problem was performed using a method combining Monte Carlo simulation with a method for performing arithmetic operations on dependent fuzzy numbers. In the article, the latter was implemented using the nonlinear programming method. As a result of this type of calculation, a fuzzy random variable was obtained, the interpretation of which may be difficult for business practitioners. Based on a critical review of existing risk measures defined for the use of hybrid data, a new method for investment decision-making was proposed. The practical part presents the investment decision-making process on the example of a tangible investment in the metallurgical sector.

Keywords: investment risk, NPV, Monte Carlo method, possibility distribution, evidence theory, p-box.

Introduction

The assumption that the future will be like the past - inherent in most forecasts, from the simplest to the most econometrically and technically complex - raises more doubts today than in past decades, especially in relation to economics and management sciences. Departing from this assumption is made possible by forecasts formulated based on expert opinions. One of the problems where it is often necessary to use expert opinions and subjective probability distributions is the effectiveness analysis of tangible investments [1].

In the literature, one can find many methods for assessing investment profitability (NPV, IRR, APV, NPVR, PI, DPP, etc.) [2]. The modern use of these methods increasingly considers the availability and scope of information held by the decision-maker. The lack of complete knowledge about phenomena stems from the complexity and variability of economic phenomena and processes occurring inside and outside the enterprise.

Until recently, probability theory was used to describe uncertainty. However, the assumptions underlying this theory are facing increasingly broad criticism [3]. This has led to the development of alternative methods for

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describing uncertainty, such as interval analysis, probability bound analysis, evidence theory, possibility theory, or credibility theory. Using these methods, it is possible in certain situations to more adequately describe the knowledge about the variables of a financial model used for effectiveness assessment, such as product prices, costs of materials, energy, labor, etc.

The adequacy of the probability distribution of the investment profitability indicator is influenced by two factors: the description of the variables' uncertainty and the method of modeling the dependencies between them [4]. It is often pointed out that the main weakness of simulation analysis is the necessity of using subjective probability distributions and the difficulties in describing statistical dependencies, which can lead to an incorrect risk assessment by the decision-maker. In the latest scientific publications, it is indicated that the barriers to simulation analysis can be reduced by using different methods of describing uncertainty within a single model [5], [6].

In real-world investment problems, the description of variable uncertainty often stems from both subjective expert opinions and historical data. We can, therefore, speak of two sources of uncertainty [7]: "aleatoric" ("objective"), which results from stochasticity and cannot be reduced by further research, and "epistemic" ("subjective"), which results from a lack of knowledge and can be reduced through additional research. In economic and managerial problems, different interpretations of probability calculus are usually used to describe both of these uncertainties.

Many researchers [5], [8], including the author of this article, believe that a purely probabilistic approach is appropriate if the uncertainty of all input parameters is of the same nature. Otherwise, a division of variables should be made according to the type of uncertainty, and then they should be processed in parallel using hybrid simulation.

The aim of this article is to present ways of making investment decisions in a situation where hybrid methods of data uncertainty processing are used.

Stages of Risk Analysis in Investment Effectiveness Appraisal

The process of risk analysis in effectiveness appraisal consists of four stages: describing the uncertainty of variables, processing the uncertainty using simulation analysis, presenting the results, and making the investment decision.

Risk analysis in the effectiveness appraisal of tangible investments begins with building a financial model. This model typically includes three categories of variables: market variables (e.g., prices, demand), investment outlays, and costs (fixed and variable). The uncertainty of variables is defined by a state space S in such a way that, at the modeling stage, the decision-maker knows that one of the states will occur, and the others will not. The decision-maker, when building the model, does not know and has no influence on which state is true and which is not. The state space can be any set, although in this article it is assumed that the state space will always be based on the set of real numbers. Subsets of the state space of nature are events. The chance of an event occurring can be determined by various measures.

Probability Theory. The measure of uncertainty of an event $T \subseteq S$ occurring is the probability measure, which has many well-known and defined properties. There are two widely recognized interpretations of probability - frequentist and Bayesian. Because in the following considerations probability theory will be used to describe uncertainty of an aleatoric nature, this article adopts the frequentist interpretation. The probability of event T will be denoted by $P(s \in T)$. The probability density function (PDF) will be denoted by ϕ, and the cumulative distribution function (CDF) by Φ, where $\Phi(s) = P((-\infty, s] \cap S)$.

Possibility Theory. Let there be a certain function π called a possibility distribution, which maps the set S to the interval $[0,1]$. The function π represents the analyst's knowledge about the possible states of the variable. The analyst can distinguish which state is the most credible, which is expected, and which occurrence would be a surprise. Example values of the function are interpreted as follows:

- $\pi(s) = 0$ – an impossible state,
- $\pi(s) = 1$ – the most credible (expected) state.

The chance of event T occurring is described by a pair of measures - necessity N and possibility Π, which are defined as follows: $\Pi(T) = \sup_{s \in T} \pi(s)$, $N(T) = \inf_{s \notin T} (1 - \pi(s))$.

A possibility distribution can be identified with a fuzzy number $\tilde{A}$ such that $\mu_{\tilde{A}} \equiv \pi$. Thanks to this, the possibility distribution can be considered in terms of α -cuts, because according to the decomposition principle, a fuzzy number can be represented as a finite sum of α -cuts:

$$\tilde{A} = \bigcup_{i=0}^{I} \alpha_i A^{\alpha_i}$$

where the α -cut is defined as: $A^{\alpha_i} = \{s \mid \pi(s) \geq \alpha_i\}$.

The main difference between probability theory and possibility theory from the perspective of risk assessment is the way uncertainty is measured. Often, the necessity and possibility measures of an event are interpreted as the bounds of an interval in which the probability of that event lies. Possibility theory is therefore better suited for describing epistemic uncertainty, as it allows for reflecting the imprecision of expert judgment [9].

In the remainder of this article, it is assumed that aleatoric uncertainty will be described by probability theory, and epistemic uncertainty by possibility theory.

As a result of the first stage, a sequence of variables is obtained whose uncertainty is described either by a probability distribution or a possibility distribution. These variables will be called risk factors. Let there be a function h_{sym}, some of whose variables are described by probability distributions and some by possibility distributions. The value of the function is the cumulative distribution function of the investment profitability indicator:

$$\Phi(npv) = h_{sym}(\ldots \Phi_k \ldots, \ldots \pi_l \ldots), npv \in NPV,$$

where NPV is the state space describing the uncertainty of the investment effectiveness indicator.

To find the value of the function h_{sym}, one should use analytical methods [10], 2D Monte Carlo simulation [8], or hybrid simulation [6]. The choice of method for determining the value of h_{sym} depends on the specifics of the variable descriptions. The nature of the considered investment effectiveness appraisal problem requires the use of the IRS (Independent Random Set) method, which belongs to the hybrid simulation group [11]. This method was originally used for the analysis of simple geological models [5].

From a technical point of view, the IRS method combines Monte Carlo simulation (sampling of random variables) and methods for performing arithmetic operations on fuzzy numbers based on Zadeh's extension principle (processing of epistemic uncertainty). The IRS+ method algorithm is as follows:

Step 1 Set the number of simulation steps to J and the number of α - cuts to I.

Step 2 Generate $J \cdot n_1$ random numbers from a uniform distribution and then use them to generate values for variables described by probability distributions. If dependencies exist between parameters, use the Cholesky matrix to account for them. This results in a sequence of vectors $\{(p_1^j, \ldots, p_{n_1}^j)\}_{j=1,\ldots,J}$.

Step 3 For each j -th vector $(p_1^j, \ldots, p_{n_1}^j)$ perform:

Step 3.1 For each i perform:

Step 3.1.1 For all possibility distributions $\pi_1, \ldots, \pi_{n_2}$ determine the bounds of the i -th α -cut: $\{(inf(\pi_k^i), sup(\pi_k^i))\}_{k=1,\ldots,n_2}$.

Step 3.1.2. Determine the bounds of the interval $[\underline{npv}^{i,j}, n\bar{p}v^{i,j}]$ by solving the following two nonlinear problems:

PROBLEM I (sup): $n\bar{p}v^{i,j} = max\, f(x_1, \ldots, x_{n_1}, y_1, \ldots, y_{n_2})$ where (i) values of vector x cannot exceed the boundary values sampled in the simulation $x_k \leq p_k^j$, $k = 1, \ldots, n_1$; (ii) values of y cannot exceed the constraints resulting from the possibility distribution for the chosen α -cut: $inf(\pi_{k_2}^i) \leq y_k \leq sup(\pi_{k_2}^i), k_2 = 1, \ldots, K$; (iii) other constraints describing dependencies between fuzzy variables.

PROBLEM II (inf): $\underline{npv}^{i,j} = min\, f(x'_1, \ldots, x'_{n_1}, y_1, \ldots, y_{n_2})$ where (i) values of vector x' cannot be less than the values obtained from solving Problem I $x'_k > x_k, k = 1, \ldots, n_1$; (ii) values of y cannot exceed the constraints of the possibility distribution for the chosen α -cut $inf(\pi_{k_2}^i) \leq y_k \leq sup(\pi_{k_2}^i), k_2 = 1, \ldots, K$, (iii) other constraints describing dependencies between fuzzy variables.

A detailed description of using hybrid simulation to determine investment effectiveness can be found in a paper by Gaweł at all [6], [11]. The result of the hybrid simulation is $J \cdot I$ intervals $[\underline{npv}^{i,j}, n\bar{p}v^{i,j}]$. Evidence theory is used to construct the risk curve based on the obtained intervals.

Evidence Theory. Let Θ be a non-empty set called the frame of discernment, such that it describes all possible NPV values of the tangible investment under study. The simulation results form a set of evidence

$$F = \{f_{i,j} \subseteq \Theta \mid f_{i,j} = [\underline{npv}^{i,j}, n\bar{p}v^{i,j}], i = 1, \ldots, I, j = 1, \ldots, J\}$$

called focal elements, relating to the true net present value of the investment. Furthermore, let there be a function m that assigns each $f_{i,j}$ an equal mass of probability equal to $m(f_{i,j}) = \frac{1}{I \cdot J}$. The measure of the truth of the hypothesis that event T occurs is described by the belief function and the plausibility function.

The belief function $Bel(T)$ defines the total belief in the evidence supporting T:

$$Bel(T) = \sum_{f_j \subseteq T} m(f_j)$$

while the plausibility function defines consistent with T.

$$Pl(T) = \sum_{T \cap f_j \neq \emptyset} m(f_j)$$

The pair $[Bel(T), Pl(T)]$ forms a belief interval, which is often interpreted as the lower and upper estimation of the probability $P(npv \in T)$.

Probability box (p-box). Let $\underline{\Phi}, \bar{\Phi}: R \to [0,1]$ be non-decreasing functions such that: $\forall_x \bar{\Phi}(x) \geq \underline{\Phi}(x)$. Then the interval

$$[\underline{\Phi}, \bar{\Phi}] = \{\tilde{\Phi}: R \to [0,1] \text{ non-decreasing } \mid \forall_{x \in R} \underline{\Phi}(x) \leq \tilde{\Phi}(x) \leq \bar{\Phi}(x)\}$$

is called a probability box (p-box), and the functions $\bar{\Phi}$ and $\underline{\Phi}$ are called the upper and lower cumulative distribution functions. The form of the function $\widetilde{\Phi}$ is not known; it is only known that it lies between $\underline{\Phi}$ and $\bar{\Phi}$. It can be shown [5] that if the NPV space is based on the set of real numbers, then $\bar{\Phi}(x) = Pl((-\infty, x])$ and $\underline{\Phi}(x) = Bel((-\infty, x])$. The set of evidence F can thus be transformed into a p-box $[Bel((-\infty, npv]), Pl((-\infty, npv])]$, which is the upper and lower bound of the set of CDFs $\Phi(npv)$. To make an investment decision based on a p-box, it is necessary to transform it into a classical cumulative distribution function.

Investment Decision Making under Stochasticity and Incomplete Information

Investment decisions can be described by the following rules:

- if $npv > 0$, the investment is economically effective and should be realized,
- if $npv \leq 0$, the investment is economically ineffective and should not be realized,
- if there are states of nature for which npv is less than zero and states for which npv is greater than zero, it is necessary to apply advanced assessment rules.

In the latter situation, the decision to accept or reject the investment depends on the investor's individual risk preferences. The fundamental problem here is defining a measure of investment risk. Three groups of methods measuring risk can be distinguished: measures of variability (dispersion of the profitability measure), measures of risk (possibility of unfavorable values of the profitability measure occurring), and measures of sensitivity (relationships between profitability measures and input parameters).

The decision criterion for a tangible investment can be written as $P(npv \in (-\infty, npv_0]) = \theta$. We can speak of two measures of risk here:

- the safety level npv_0 of the investment for an assumed level θ,
- the probability of not achieving the assumed aspiration level npv_0.

In contrast to classical simulation analysis, the result of hybrid simulation is a p-box $[Bel((-\infty, npv])$, $Pl((-\infty, npv])]$. Both risk measures are represented in this situation by intervals, which complicates their interpretation by the decision-maker. It is therefore necessary to transform the p-box into a cumulative distribution function of npv.

According to the classical approach defined by Knight, three conditions under which a decision-maker makes an investment decision can be distinguished:

- **Situation 1 (Certainty)** - a closed set of alternatives, consequences defined deterministically.
- **Situation 2 (Risk)** - The investor knows the risk curve (probability distribution).
- **Situation 3 (Uncertainty)** - The investor does not know this distribution.

This division, despite its theoretical elegance, is a significant simplification of reality. Usually, in the appraisal of tangible investments, one deals with a situation between uncertainty and risk, because it is very rare to know nothing about the probability, but it is equally rare to know it exactly. As Ellsberg's experiments show [12], people's choices in situations of risk may differ from choices where information about the chances is imprecise. Studies show that in such situations, choices can contradict utility theory.

In Ellsberg's opinion, an intermediate state should be introduced between complete ignorance about probability and full knowledge of the distribution. This state was called ambiguity. Ambiguity has its sources in the low reliability of information or conflicting information.

The mathematical reflection of ambiguity is a situation in which the decision-maker deals with risk described using upper and lower cumulative distribution functions (a p-box). The decision-maker aims to reduce ambiguity and thus find a transformation that will allow the p-box to be converted into a curve illustrating risk. To achieve this goal, the decision-maker can adopt many strategies.

The wider the interval, the harder it is to make an investment decision. The perception of this interval depends largely on the investor's risk aversion. In extreme cases, the choice of a conservative investor will be the value of the upper CDF, and an optimistic one the lower CDF. However, in reality, very few investors have such an aggressive approach to risk. Below are several strategies for constructing a risk curve based on the upper/lower CDF pair.

Bayes-Laplace Criterion The first approach was proposed by Smets [13] under the name "pignistic transformation". It is based on the Bayes-Laplace decision criterion, according to which the set of evidence F is a set of possible and indistinguishable (from the point of view of utility theory) intervals of NPV values. The sets of evidence can thus be treated as sets of payoffs to which an equal probability of occurrence equal to $m(f_{i,j}) = \frac{1}{I \cdot J}$ is assigned. The expected payoff value is obtained by transforming the evidence $f_{i,j} = [\underline{npv}^{i,j}, \bar{npv}^{i,j}]$ into a uniform distribution $\varphi_{uniform}(\underline{npv}^{i,j}, \bar{npv}^{i,j})$. The form of the distribution $\Phi_{bet}(npv)$ is determined from the equation:

$$\Phi_{bet}(npv) = \sum_{npv \in f_{i,j} \subseteq F} \frac{m(f_{i,j})}{| f_{i,j} |}.$$

This approach does not consider the investor's risk aversion.

Hurwicz Criterion Ambiguity [14] expresses uncertainty about the knowledge of probability and can be defined as:

$$Ambiguity(\kappa) = Pl^{-1}(\kappa) - Bel^{-1}(\kappa), \kappa \in [0,1].$$

Ambiguity is an intermediate state between complete ignorance and knowledge of probability. Using the Hurwicz criterion, and thus treating the Bel and Pl curves as extreme scenarios, Jeffrey defines the ambiguity distribution as:

$$\Phi_{amb}^{-1}(\kappa) = \lambda Pl^{-1}(\kappa) + (1 - \lambda) Bel^{-1}(\kappa), \kappa \in [0,1],$$

where λ takes values from the interval 0 to 1 and is called the decision-maker's pessimism (optimism) index.

Credibility Distribution The third approach [15] uses credibility theory proposed by Liu. He proposes approximating $\Phi(npv)$ with the credibility distribution:

$$\Phi_{cred}(npv) = \frac{Bel((-\infty, npv]) + Pl((-\infty, npv])}{2}$$

Dubois extends the measure proposed by Liu with a risk aversion coefficient β:

$\Phi_{cred}(npv) = \beta \cdot Bel((-\infty, npv]) + (1 - \beta) \cdot Pl((-\infty, npv])$ Ambiguity can thus be interpreted as a safety interval $[\underline{npv}_0, \bar{npv}_0]$ for an assumed level θ, and credibility as a probability interval $[\underline{\theta}_0, \bar{\theta}_0]$ for an assumed aspiration level npv_0.

Using Hybrid Simulation in the Effectiveness Appraisal of a Tangible Investment

The differences between approaches are presented using the example of an effectiveness appraisal for an investment project concerning the construction of a department for organic coating of galvanized sheets, implemented in a metallurgical industry enterprise.

The effectiveness of investment projects in the steel industry mainly depends on the following parameters: demand, raw material prices, sales prices of products, material consumption rates, and the value of investment outlays. In the profitability and risk analysis, it was assumed that the decision-maker does not have complete knowledge about the following parameters: apparent consumption of products, sales of individual products offered by the steelworks, prices of products offered by the steelworks and semi-finished products for their production, material consumption rates, value of investment outlays. Furthermore, it was assumed that the remaining parameters in the appraisal take deterministic values. Sales were expressed as the product of the forecasted apparent consumption of individual products manufactured by the analyzed enterprise and the forecasted market share.

The diagram of the production process carried out by the analyzed enterprise is shown in Fig. 1.

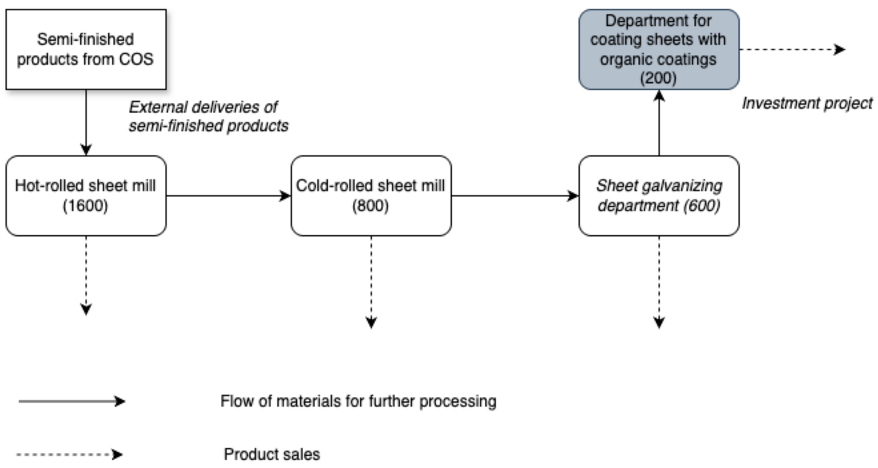

Fig. 1. Diagram of the analyzed production process. Source: Own elaboration

Table 1 presents the possibility distributions and parameters of the probability density function (mean - m, standard deviation - σ) defining the forecasts of parameters for calculating the net present value for year $t = 0$ of the project's economic life cycle.

Table 1. *Trapezoidal fuzzy numbers defining the prices of products and input semi-finished products and material consumption rates, as well as the parameters of the probability density function characterizing the apparent consumption of steel products.*

Category	Item	Value (Trapezoidal Fuzzy Numbers, USD/t or t/t) / (Mean, thous. t; Std. Dev., thous. t)
Prices	Semi-finished products from COS	(440.3; 445.0; 450.7; 460.3)
	Hot-rolled sheets	(640.0; 652.8; 683.2; 699.2)

	Cold-rolled sheets	(786.4; 700.8; 732.8; 750.4)
	Galvanized sheets	(772.8; 788.8; 825.6; 844.8)
	Organically coated sheets	(1,036.8; 1,057.6; 1,107.2; 1,128.0)
Material Consumption Rates	Semi-finished COS – hot-rolled sheets	(1.058; 1.064; 1.075; 1.078)
	Hot-rolled sheets – cold-rolled sheets	(1.105; 1.111; 1.124; 1.130)
	Cold-rolled sheets – galvanized sheets	(1.010; 1.020; 1.026; 1.031)
	Galvanized sheets – organically coated sheets	(0.998; 0.999; 1.000; 1.001)
Apparent Consumption	Hot-rolled sheets	(2,704.0; 117.5)
	Cold-rolled sheets	(1,162.3; 51.4)
	Galvanized sheets	(1,147.9; 52.4)
	Organically coated sheets	(708.4; 30.8)

To simplify calculations, it was assumed that the apparent consumption of steel products in subsequent years will increase by 1.5% annually. At the same time, product prices and material consumption rates will remain constant over the analyzed period. The value of fixed costs was assumed as a deterministic quantity at the level of USD 315,090 thous./year. The adjusted processing costs for the analyzed products were also set as deterministic values (Table 2).

Table 2. Adjusted processing costs for individual product assortments produced by the analyzed manufacturer.

Product	Hot-rolled sheets	Cold-rolled sheets	Galvanized sheets	Organically coated sheets
Adjusted unit processing cost, USD/t	28.4	28.0	116.7	175.3

Market shares for individual steel products are presented in Table 3.

Table 3. Market shares of the analyzed manufacturer for individual product assortments.

Hot-rolled sheets	Cold-rolled sheets	Hot-dip galvanized sheets	Organically coated sheets

42.5%	40.0%	46.0%	45.0%

The value of investment outlays was described as a possibility distribution (42,000; 46,000; 50,000) in thous. USD.

Using the data described above, a simulation analysis was conducted using the Monte Carlo method and the hybrid method. To perform the Monte Carlo analysis, all variables described by possibility distributions were transformed into probability distributions, using the method described in.

The results of both simulations and the risk curves are presented in Fig. 2. The solid lines mark the upper and lower cumulative distribution functions (p-box) of the investment profitability indicator, and the dotted line marks the stochastic simulation. The graph also shows the curves obtained from the transformations described in the article. The following parameters were assumed for the transformations: $\lambda = 0.5$ in Φ_{amb} and $\beta = 0.5$ in Φ_{cred}.

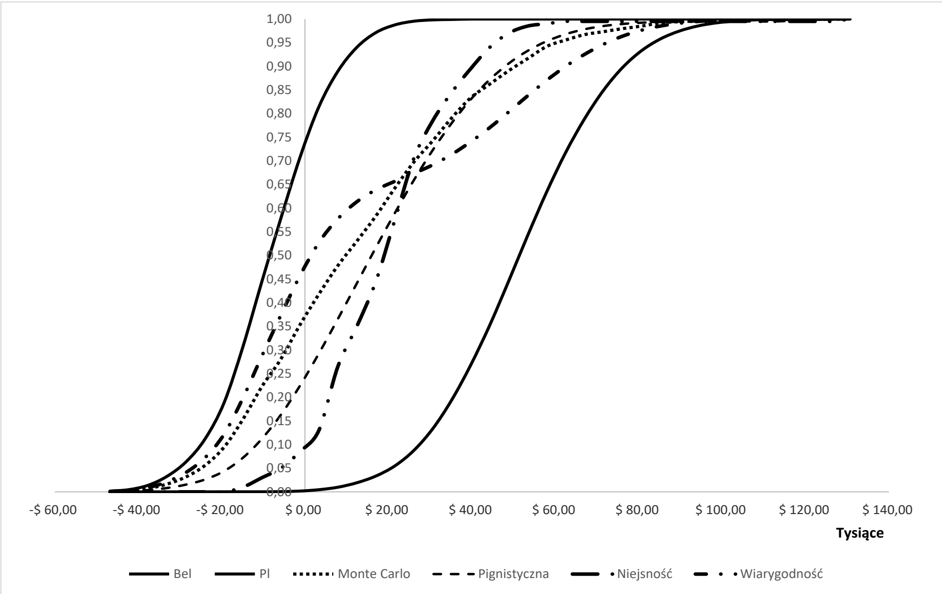

Fig. 2. Comparison of results from stochastic simulation and hybrid simulation. Source: Own elaboration

Table 4 presents a comparison of risk indicators for individual decision approaches.

Table 4. Comparison of risk indicators $P(npv \leq npv_0)$ and npv_0 for an assumed θ level, for $npv_0 = 0$ and $\theta = 0.9$.

Simulation / Transformation	npv0 (in thous. USD) for θ=0.9	P(npv≤0) for npv0=0
Φ_{bet}	9,027.37	0.37
Φ_{amb} ($\lambda = 0.5$)	39,745.52	0.35
Φ_{cred} ($\beta = 0.5$)	67,050.55	0.35
Monte Carlo Simulation	76,152.22	0.37
p-box (Hybrid Simulation)	$[-52{,}348.90; 47{,}709.49]$	[0; 0.7]

The probability $P(npv \leq 0)$ obtained from the stochastic simulation is 0.37, while based on the hybrid simulation, it can be stated that this probability lies within a wide interval between 0 and 0.7. The distance between the upper and lower bounds of the interval stems from the uncertainty in the description of the input variables. The results of the stochastic and epistemic simulations can thus be interpreted jointly as follows: the value of the probability $P(npv \leq 0)$ is 0.37, but the error from subjective expert assessment and stochasticity causes the actual probability value to be in the interval [0; 0.7]. It is up to the decision-maker to assess the significance of this error. If they consider it too large, they can order additional measurements. This shows the advantage of the hybrid approach over the purely probabilistic one when dealing with parameter uncertainty arising from randomness and imprecision. Unlike the purely probabilistic approach, the decision-maker receives an interval whose width illustrates the imprecision of the risk measurement resulting from the estimates adopted by the analysts.

For various reasons (cost, time), performing additional research is not always possible. In this case, instead of using stochastic simulation, the decision-maker can make a decision using one of the transformations presented above. When making an investment decision based on the risk indicators estimated from the curves resulting from the transformation, one must consider the characteristics of that transformation. The individual CDFs consider different types of investors. Specifically:

- the transformation to Φ_{bet} assumes that the decision-maker's preferences are unknown,
- the transformation to the ambiguity distribution Φ_{amb} assumes that the investor makes a decision based on the expected payoff value; the parameter λ is used to assign weights to the boundary values within the focal elements,
- the transformation to Φ_{cred} assumes that the key criterion guiding the investor is the probability of a given state of the effectiveness indicator occurring. The risk aversion index β assigns weight to the boundary probabilities for a given payoff.

In the authors' opinion, the value of the $P(npv \leq 0)$ indicator should be estimated based on the credibility distribution, and the safety level based on the ambiguity distribution. In the discussed example, this would mean that $P(npv \leq 0) = 0.35$, and $npv_0 = \$39{,}745.5$.

Such an approach highlights the second benefit of using hybrid simulation, which is the ability to include the investor's characteristics in the results.

Using hybrid simulation also allows one to assess the extent to which the CDF (Cumulative Distribution Function) obtained from Monte Carlo simulation may influence the investor's decision. In an ideal situation, the curves: Φ_{amb}, Φ_{cred} (for $\lambda = 0.5, \beta = 0.5$) and the stochastic simulation should overlap. If the Monte Carlo CDF is below/above the curves, it means that the results obtained with it may be burdened with some optimism or pessimism.

To determine these curves, the concept of stochastic dominance was used. Function F shows first-order stochastic dominance (FSD) over G when $\forall x F(x) \leq G(x)$. Function F shows second-order stochastic dominance (SSD) over G when $\forall x \in R \int_{-\infty}^{x} (F(t) - G(t))dt \leq 0$. The following situations are of interest:

- $\max_{\lambda,\kappa} \rightarrow \Phi_{amb}(npv) \leq \Phi_{MonteCarlo}(npv)$ (FSD)
- $\max_{\beta,npv} \rightarrow \Phi_{cred}(npv) \leq \Phi_{MonteCarlo}(npv)$ (SSD)

The solutions to these problems are $\lambda = 0.38$ and $\beta = 0.35$, respectively. It can therefore be stated that the probabilistic description of aleatoric and epistemic uncertainty introduces additional information into the model, which causes the simulation results not to overlap with the neutral curves: Φ_{amb}, Φ_{cred} (for $\lambda = 0.5, \beta = 0.5$). Monte Carlo simulation thus introduces information into the model that does not result from the nature of the variables but may affect the decision-maker's decisions.

Summary

The article presents the results of pilot studies on investment decision-making when possibility distributions and probability distributions are used to describe input variables. Based on real data, the benefits of using hybrid simulation in conjunction with stochastic simulation were presented. In the next stage, the research will focus on the following problems:

- using entropy to build indicators for assessing investment effectiveness,
- extending the transformations described in the article to include the use of prospect theory.

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